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SIMULTANEOUS COLOR MEASUREMENTS OF ILLUMINANTS BY TRICHROMATIC AND MONOCHROMATIC ANALYSIS

By Herbert E. Ives

Introduction

Two different color-mixture methods of color measurement have been employed in measuring the colors of illuminants. In the first of these, *the trichromatic method*, three primaries, usually a red, a green and a blue, are mixed to form a match with the color under measurement, and their relative proportions constitute the desired measure of the color. It is the usual custom to take as the unit for each primary the amount used to mix with the others to make white, but other units may be used, such as equal luminous values of the three primary colors. A large number of choices of primaries is possible; measurements in terms of one set may be transformed by algebraic processes into terms of any other. Reduction of all trichromatic analyses to some commonly-agreed-upon set of primaries is necessary for intercomparing their findings; the "fundamental sensations" as determined by Koenig constitute a set of primaries whose fundamental character recommends their adoption as the standard trichromatic reference frame. The second color mixture method of color measurement, *the monochromatic method*, matches colors by mixtures of some standard color of low saturation, with any one of a continuous series of colors of high saturation. For the color of low saturation it is common practice to take "white"; for the series of high saturation colors, the spectrum. The units ordinarily chosen are *equal luminous values* of white and spectrum color; the measurements are expressed in terms of

the *luminosity* of the measured color, its *spectrum hue*, in wave-length units (e.g. Angstrom units), and the proportionate part of its total luminosity which is contributed by the spectrum light in the mixture, called its *saturation* or *purity*.

Measurements made by these two methods should of course be capable of correlation. Such correlation has however been difficult for several reasons. Leaving aside the differences in the color vision of observers, which offer obstacles to the correlation of observations made even by the same method, three principal difficulties in comparing measurements made by the two methods may be noted. The first is that the "whites" used for reference have not been definitely the same; "sunlight," "average daylight," or a black body at some temperature, such as 5000°, having been used in various sets of observations and computations. The second is that the measurement of the relative luminous values of the color which is the object of measurement, and of its constituent white and spectrum light, which is one step in the monochromatic measurement, is not only difficult, since it is subject to all the uncertainties of color-difference photometric setting, but has not heretofore been made by such methods or under such conditions as to insure its agreement with any well established luminous efficiency curve of the spectrum, such as must be used in any computations. The third is that the algebraic transformation processes necessary for the intercomparisons have not been available in easily applicable form.

It is the purpose of this paper to record and discuss a series of measurements of the colors of illuminants made simultaneously by the two methods above discussed, under such conditions that the obstacles just listed were largely overcome. Taking up in order the means by which these conditions were brought about: the first to claim attention is the reference white used. This was a 5000° black body, as produced artificially by the use of the spectrum template screen described some years ago under the title of "An Apparatus for the Spectroscopic Synthesis of Color."[1] The 5000° black body color thus produced was recorded and preserved by a secondary standard consisting of a piece of daylight glass combined with a tungsten lamp at a definite efficiency. This secondary standard was in turn matched for the present investigation by a large high-efficiency tungsten lamp and a sheet of the same kind of daylight glass.

[1] Ives and Brady, Jnl. Franklin Institute, July 1914, p. 89.

The color difference photometry was handled by means of the color-match photometer described in a previous article.[2] This makes a trichromatic match in terms of a yellow, a green and a blue which are just saturated enough so that the ordinary artificial illuminants may be measured. It records luminous values in accordance with the spectral luminous efficiency curve recommended by the writer, and at the same time provides color measurements in terms of the mixing proportions of its primaries; these color measurements will be used as the trichromatic series in the intercomparison of methods.

The algebraic transformations which are needed have recently been put in convenient graphical form, using the 5000° black body as the white, and the color-match-photometer mixing colors as the set of "working primaries."[3] These primaries with their included triangle, (whose sides are divided into equal divisions) and the spectrum, have been projected upon the fundamental sensation triangle, upon which are also projected the loci of equal luminous steps between white and spectrum. The complete diagram is given in the previous paper on the color match photometer (Fig. 13 of that paper) and a portion of it, with experimental values as derived in this study, is shown in Fig. 1. With this digram available it is possible not only to plot both the trichromatic and monochromatic analysis results of illuminant measurements in the one plane, but, what is of greater ultimate importance, to determine from the measurements how close is the agreement between the fundamental data used in the computations of the transformation processes. Specifically the data in question are the fundamental sensations as determined by Koenig and modified by the present writer,[3] and the luminous efficiency curve of the spectrum, as determined in investigations by the writer and others.[4]

APPARATUS AND METHODS OF OBSERVATION

The complete arrangement of apparatus for performing the simultaneous measurements is shown diagrammatically in Fig. 2. The illuminant to be measured is placed upon a photometric carriage X movable on the track T. Its light falls upon the face of the white sector disc R and is observed by the eye at E, through a 5 mm diameter artificial pupil. This light is matched by light from the color match

[2] Ives, J. O. S. A. & R. S. I., March, 1923.
[3] Ives, Jnl. Franklin Institute, January, 1923, p. 23.
[4] Ives, Jnl. Franklin Institute, August, 1919, p. 217.

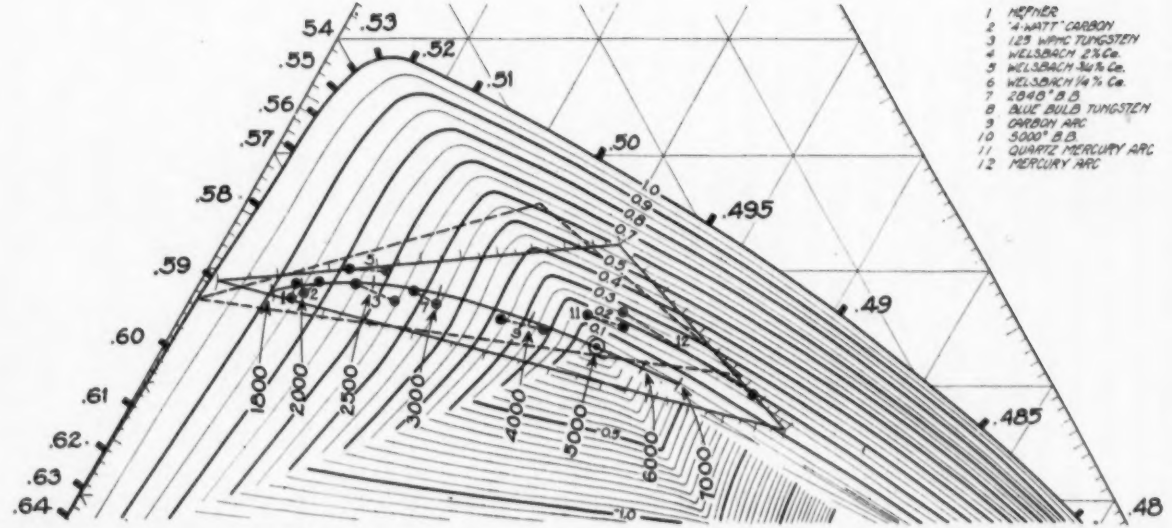


FIG. 1. *Computed and experimental color measurements plotted in Koenig sensation color triangle. The inner full line triangle is the color triangle enclosed by the color match photometer primaries. The heavy dots are the trichromatic measurements as made with the color match photometer. The circles and dots are the monochromatic measurements, as plotted on the series of computed curves lying between the spectrum and the white center. The dashed triangle connects the color match photometer primaries as measured by the monochromatic analysis and plotted on the white-to-spectrum co-ordinate system.*

photometer C which falls upon the diffusion chamber D of that instrument, and is observed at E through the prism B_1 and the biprism B_2. A diaphragm F at the biprism reduces the field to a disc 2 degrees in diameter. The monochromatic light is furnished by the spectrometer P, an image of whose eye-slit is projected by means of the lens L_2 upon the artificial pupil at E. The appearance of the artificial pupil when examined from some distance along the optical axis is as shown in the insert to the left. It will be noted that this optical arrangement permits the observation, when the eye is centered, of the color match

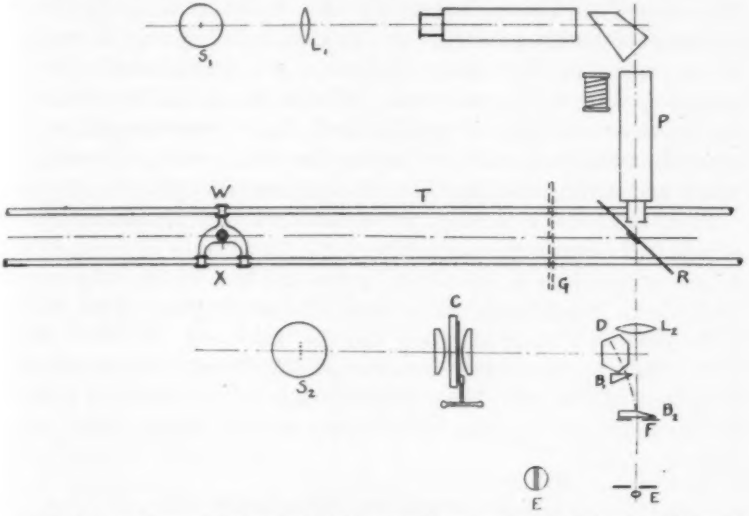


FIG. 2. *Arrangement of color-match photometer and spectrometer for performing trichromatic and monochromatic measurements upon the same illuminants.*

photometer field at its proper working brightness (25 meter candles illumination) at the same time that a narrow slit is used with the spectrometer, furnishing highly monochromatic light. The intensity of the monochromatic light is varied by means of a resistance in series with the helical tungsten filament at S, whose image is focussed on the collimator slit by the lens L_1. The wave-length is altered by turning the wave-length drum of the spectrometer by means of a long flexible shaft. White light to mix with the spectrum light is furnished by placing a special tungsten lamp W, in the movable photometer carriage, and interposing the piece of daylight glass G between W and the sector disc R.

The method of observation is as follows: The illuminant to be measured is placed at X, and moved along the track T until an approximate intensity match is obtained in the observing field with the light from the color match photometer. The latter is then adjusted until the two fields match in color, and the intensity match is made perfect. Then holding the illuminant's color by the color match photometer field the white light is substituted for the illuminant and the disc R spun at high speed. The field formerly illuminated by the test lamp is now illuminated by a mixture of white and monochromatic light. The relative and absolute amounts of these lights are varied by moving W along the photometer track and by altering the current through S_1, at the same time that the wave-length of the monochromatic light is shifted up or down the spectrum. When a match has been obtained, the spectrometer light is extinguished, and a measurement of the white illumination is made by setting the color match photometer to white, and moving the lamp W until a photometric match is obtained.

From this series of settings, *all of which are made at color match*, we obtain trichromatic measurements of the illuminants in terms of the mixing proportions of the yellow, green and blue of the color match photometer, and monochromatic measurements in terms of the mixing proportions of a spectrum color and the white, e.g. the 5000° black body. Such measurements on twelve artificial illuminants are recorded in Table 1 and in Fig. 1. All measurements are the means of at least five settings; in every case the observer was the writer, whose color vision is normal.

THE TRICHROMATIC MEASUREMENTS

The trichromatic measurements have but little interest as long as they are expressed only in terms of the arbitrarily selected yellow, green and blue of the color match photometer. Such absolute value as they have is made evident when they are expressed in terms of some fundamental set of primaries. They are accordingly transformed by the graphical method described in a former paper[3] into the system of which the primaries are Koenig's fundamental sensations. This method consists in projecting the color triangle of the color match photometer, with the various measurements plotted as points within it, upon the color triangle of the fundamental sensations; considering the points representing the measurements as lying in the sensation triangle, we can at once read off their coordinates in terms of sensations. Color match photometer values as read are tabulated in Table

1, columns, 3, 4 and 5; and the derived sensation values in columns 6, 7 and 8.

A check of a certain sort is obtained on these sensation values by means of those illuminants whose radiation corresponds to that of a black body. Thus the smooth curve marked in temperatures gives the sensation values for the series of black bodies from 1800° to 7000°, as computed from their radiant emissions as given by the Planck equation. It will be noted that the Hefner lamp (color temperature about 1840°) the "4 watt" carbon lamp (2080°), the 1.25 wphc tungsten lamp (2330°), the high efficiency tungsten (2848°) and the white (5000°) fall very closely upon the black body line. Since the color match photometer closely copies the spectral energy distribution of these sources, this agreement of computed and observed values shows that the various measurements and computations involved in the development of the color match photometer have been carried through without substantial error, so that the sensation values of the illuminants of other than black body type may be relied upon as correct.

THE MONOCHROMATIC MEASUREMENTS

In Table 2, columns 4 and 5 are recorded the wave-lengths and purities of eight illuminants as directly measured. With the exception of the mercury arcs and the Welsbach mantle the spectrum hues all lie between 5840 and 5910 Angstrom units, the chief differences in color being due to differences in saturation, namely 22 for the carbon arc and 86 for the Hefner.

In columns 2 and 3 are given color measurements of the same sort made by L. A. Jones,[5] using "sunlight" as the white. This should be not far from the 5000° black body, so that a fairly close agreement between Jones' values and these new ones should be expected, provided his measurements of luminosity were on substantially the same photometric scale. Comparison of the two sets of data does in fact show on the whole a good agreement, the biggest difference being in the assigned purity values for the carbon arc (22 and 33) which may well be due to different amounts of light from crater and arc in the two cases. For the redder black body illuminants Jones assigns longer wave-length spectrum hues; for the carbon and mercury arcs shorter wave-length ones than the present study indicates. The degree of agreement of the two sets of measurements indicates that the new monochromatic-analysis values are substantially correct.

[5] Jones, Trans. Ill. Eng. Soc., 1914.

INTERCOMPARISON OF TRICHROMATIC AND MONOCHROMATIC MEASUREMENTS

For purposes of intercomparison it is necessary to reduce all the measurements to one common reference frame. This is done in Fig. 1 where the background is the Koenig fundamental sensation color triangle, as modified by the writer;[3] the small superposed triangle shown by full lines is the color match photometer color triangle as computed from the spectral energy distributions of its primaries, the curves roughly parallel to the spectrum line (marked in wave-lengths) indicate various purities, measured outward from the 5000° black body, the reference white for all the measurements, as calculated by methods described in a former paper.[3] The solid black dots are the color match photometer measurements, the circles and enclosed dots are the monochromatic measurements, the corresponding points being connected by dashed lines.

The first colors to claim attention are the yellow, green and blue of the color match photometer. Their values computed from their spectral energy distributions by the use of the sensation curves, are the vertices of the small full line triangle. Their positions as measured by the monochromatic analysis are given by the vertices of the small dashed line triangle. The computed monochromatic and trichromatic sensation values are given in Table 1, columns 6 to 10; the observed monochromatic values, and the sensation values derived therefrom by reference to the underlying sensation triangle are shown in columns 11 to 15. It is obvious at once that there is a serious discrepancy between the two sets of measurements: the monochromatic analysis indicates in every case a longer wave-length of the spectrum hue.

Turning to the illuminant measurements which are similarly tabulated, we note first of all that the monochromatic analysis values all share the same wave-length shift already noticed for the color-match-photometer primaries. In addition to this wave-length shift the purity is in practically every case considerably less as measured by the monochromatic analysis, than as indicated from the color match photometer readings. The net result is a revolution of all monochromatic-analysis points in the counter-clockwise direction, and a contraction toward the center. These shifts are most clearly indicated by the plot of Fig. 1, but are in addition tabulated fully in Table 1, where the exact amounts of the discrepancies between the two sets of measurements are expressed both in terms of wave-length and saturation, and

TABLE 1. *Summary*

	Source	Color Match Photometer								Monochromatic Analysis				
		Measured			Derived Sensations			Derived		Measured		Derived Sensations		
		Y	G	B	RS	GS	BS	λ	% Purity	λ	% Purity	RS	GS	BS
1	Hefner	.884	.10	.016	.541	.389	.07	5890	87	5910	86	.550	.375	.075
2	4 watt carbon	.84	.14	.02	.522	.39	.088	5875	83	5900	84	.540	.379	.081
3	1.25 wphc Wo	.77	.19	.04	.493	.389	.118	5860	76	5884	65.5	.480	.370	.150
4	Welsbach 2% Ce	.76	.24	0	.492	.399	.109							
5	Welsbach ¾% Ce	.69	.31	0	.468	.401	.131	5840	79	5824	71	.471	.399	.130
6	Welsbach ¼% Ce	.63	.31	.06	.438	.388	.174							
7	2848° T. C.	.68	.24	.08	.448	.385	.167	5840	63	5869	56	.441	.371	.189
8	Blue Bulb tungsten	.58	.30	.12	.452	.374	.174							
9	Carbon arc	.57	.23	.20	.396	.356	.248	5860	37	5841	22	.369	.348	.283
10	5000° black body	.40	.23	.37	.333	.331	.336				0	(.333)	(.333)	(.333)
11	Quartz Mercury arc	.32	.43	.25	.304	.347	.349	4940	17	5068	15	.329	.36	.311
12	Mercury arc	0	.25	.75	.235	.292	.473	4830	19	4956	25	.30	.363	.337
	Yellow primary	100	0	0	.599	.3884	.0126	5901	99	5931	100	.621	.375	.004
	Green primary	0	100	0	.273	.421	.306	4985	61	5078	63.2	.318	.459	.223
	Blue primary	0	0	100	.225	.260	.515	4780	12	4856	25	.237	.310	.453

in terms of sensations. It is to be noted that the wave-length discrepancies appear disproportionately large in the diagram in the green and blue region of the spectrum because of the exaggerated extension of the spectrum in the color triangle in this region. Also, it may be remarked, the differences as expressed in sensation units do not, as shown by comparing the two "derived sensations" columns of Table 1, appear excessive except in the blue region (mercury arc). However the values indicated by either method for the proper settings in the other would result practically everywhere in colors quite too far off for serious precision color measurement.

TABLE 2

Light Source	JONES White = Sunlight		IVES White = 5000° Black Body	
	λ	Purity %	λ	Purity %
Hefner	5930	86	5910	86
Carbon lamp 4 wphc	.....	.....	5900	84
3.8 wphc	5915	75	.....	.....
Tungsten 1.25 wphc	5880	65	5884	65.5
Tungsten 2848° T.C	.....	.....	5869	56
Tungsten .5 wpc	5845	55	.....	.....
Mercury arc	4900	30	4956	25
Quartz Mercury arc	.....	.....	5068	15
Carbon arc	5835	33	5841	22
Welsbach ¾% Ce	.....	.....	5824	71
Acetylene	5855	64	.....	.....
Pentane	5920	85	.....	.....

DISCUSSION

The cause of the disagreement between the two kinds of measurement is of course to be looked for in disagreements among the fundamental visual data used, and between the fundamental visual data and the characteristics of the eye used in the measurements. Taking the measurements as they stand it is indicated that (1) the fundamental sensation curves must be slightly altered in the yellow, and considerably altered in the blue of the spectrum, in shape and crossing points, and (2) the purity steps must be more nearly equal in the white to yellow region, that is, that a luminosity curve must be used which lies farther toward the blue. In view of the fact that Koenig's curves were obtained under different conditions of field size and brightness from those used to get the luminous efficiency curve, with far fewer observers,

it is hardly to be expected that a close agreement of the data will be found. Furthermore there is reason to doubt the accuracy of Koenig's data in the blue region of the spectrum; the alteration of the blue primary, as performed by the writer, is perhaps only a partial correction, as it involves a change in the crossing points of the three curves, in just the region, where as above indicated, the experimental results show the transformed curves to be unsatisfactory. Whether all or only part of the fundamental data are in disagreement with the characteristics of the particular observer's eye is not directly indicated by these measurements, but other data suggest that the luminous efficiency curve used is not far from that of the writer, so that the chief source of discrepancy is in the color sensation data.

In order for monochromatic and trichromatic measurements to agree when transformed into each other it is necessary to have complete luminosity and color sensation data made under the same conditions for the eye by which the measurements are made. Such complete data are not now available, so that the transformation processes which have been developed can at present only represent an ideal; how short it is of attainment is shown by the measurements here made.

A word may be said on the general subject of color measurement and specification as affected by the subject matter of this paper. As previously emphasized by the writer the only unambiguous measurement of color is the measurement of spectral distribution of energy. This measurement may then for purpose of simple description and specification be transformed either into terms of three sensations or into terms of hue, purity and luminosity, using sensation and luminosity data much more truly average or representative than are now at hand. As to which three-coordinate system is preferable, there is much to be said for the monochromatic system, since the spectrum, which is used as the reference scale of hues, has a unique physical definiteness. It will probably be much easier to educate the color user to the hue scale defined by the spectrum than to a three sensation scale.

Attention may be called to the fact that the transformation from spectrophotometric data to hue, purity, and luminosity may be made through *any* set of three color mixture curves of the spectrum, that is, it is not necessary to transform these so that the primaries shall be those of the two classes of color blind or even in such a manner as to have no negative values. The determination of a set of color mixture curves of the spectrum, using any reasonable primaries (such as those chosen by Maxwell), under approved photometric conditions and by a large group

of observers, would be a most important contribution to practical colorimetry. Monochromatic analysis values as computed from spectrophotometric data through such a set of curves, while they could not be expected to exactly check any chance observer's direct measurements, should check much closer than those here recorded, and would accurately describe the color for the average eye.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE AND TELEGRAPH CO.,
AND THE WESTERN ELECTRIC COMPANY, INC.
NOVEMBER 27, 1922.

NOTE ON LLOYD'S MIRROR

By J. B. Green

An observer working for any length of time with Lloyd's single-mirror interference experiment[1] is struck by two singular phenomena: first, a change in the regularity of the fringe spacing, i.e., a gradual broadening of the fringes, at the end of the interference pattern next the mirror; second, a periodic variation in the visibility of the interference fringes, similar to a beat phenomenon.

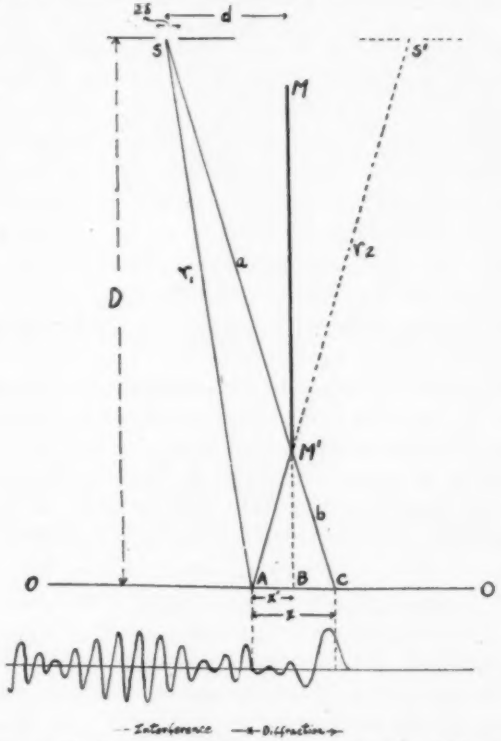


Fig. 1. *Lloyd's mirror, showing intensity and spacing of fringes.*

Broadening of the Fringes. This occurs in the region AC (Fig. 1) and since it is beyond OA, where interference occurs, the explanation is not obvious. However, on closer study of these fringes,

[1] See e.g., Wood, "Physical Optics," p. 185.

by moving the end M′ of the mirror nearer to OO′, the plane of observation, the fringes are found to be due to diffraction by the edge M′, of the light from S, the slit. But, if this is the case, it still remains to be explained why the diffraction fringe at A should be of the same width as the interference fringe, and in the same phase, so that the two systems match on to each other without a break.

We may express the distance x, from the edge of the geometrical shadow, C, to the point A, by

$$x=\frac{2bd}{a}=\sqrt{\frac{b\ (a+b)}{a}\ (2n+p)\lambda}, \tag{1}$$

where d is the average slit distance, n is an integer and p runs from -1 to $+1$, and expresses the phase of the intensity at A. (For $p=0$, it is a minimum; for $p=1$, a maximum.) The width of the diffraction fringe in which A is situated is given by the difference between (1) and a similar expression in which n is replaced by $(n+1)$. If this difference be expanded according to the binomial theorem, it may be shown that the width of the fringe is

$$\frac{(a+b)\lambda}{2d}=\frac{D\lambda}{2d}, \tag{2}$$

provided the quantity $\frac{a(a+b)\lambda}{2\ bd^2}$ is smaller than 0.2. This will be true if n is more than 3 or 4 which is the case in this experiment. The expression (2), the width of the diffraction fringe, is thus seen to be the same as the width of an interference fringe in this case.

It remains to be shown, then, that the phase of the intensity of the diffraction and interference systems must be the same at A, in order that the transition be continuous. Now, we may express the distance AB as[2]

$$x'=\frac{x}{2}=\frac{bd}{a}=\frac{(k+q)\ (a+b)\lambda}{2d}, \tag{3}$$

where k is an integer, and q runs from $-\frac{1}{2}$ to $\frac{1}{2}$ (For $q-=0$, the intensity is a minimum; for $-q=\frac{1}{2}$, it is a maximum). If we compare this expression with (1), we see that

$$n=k, \text{ and } p=2q,$$

which is the condition that the intensity of the two systems be in the same phase at A, for the variation of p is twice as rapid as that of q when going from a maximum to a minimum of intensity.

[2] See, e.g., Wood, "Physical Optics," p. 130-1.

PERIODIC VARIATION OF VISIBILITY OF FRINGES:—At first, one is likely to confuse this with the variation in visibility of the fringes as seen in the Michelson interferometer due to the fact that the sodium D-line is a doublet. But the period of the latter variation is about 1000 fringes, while in the present experiment the period depends on the distance d and the slit width and is usually of the order of ten fringes. By changing the distance d, leaving the slit-width fixed, the whole pattern seems to slide over the field, altering the width of the fringes, but leaving the maxima and minima in the same places. However, if the slit-width be changed, the distance d remaining constant, the width of the fringes stays the same, while the visibility maxima and minima appear to slide over the field of vision. The latter positions are thus seen to be a function of the slit-width alone. The following consideration will serve to explain these facts.

The parts of the slit nearer the mirror give rise to a fringe system whose spacing is closer than that caused by the parts farther from the mirror, and the overlapping of the different systems will clearly cause the beat phenomenon observed. Since the waves sent out from the different parts of the slit are non-coherent, it is necessary to add their energies and not their amplitudes. Assuming that the waves are cylindrical, the amplitude, at a point on OO′, of the electric vector due to an element of slit ds and its coherent image at distances s from the mirror, is proportional to the square root of the sum of the squares of the real and imaginary parts of

$$U=\frac{e^{-i\frac{2\pi}{\lambda}r_1}}{\sqrt{r_1}}-\frac{e^{-i\frac{2\pi}{\lambda}r_2}}{\sqrt{r_2}} \tag{4}$$

(the minus sign because of the change of phase on reflection), where

$$r_1=\sqrt{D^2+(x'-s)^2} \text{ and } r_2=\sqrt{D^2+(x'+s)^2}.$$

Now we may consider $\sqrt{r_1}$ and $\sqrt{r_2}$ equal and constant (for small changes of x' and s) in the denominators of the factors in (4), since the phases in the exponentials are the determining quantities for the positions of the maxima and minima of energy. Expanding r_1 and r_2, remembering that $(x'-s)$ and $(x'+s)$ are small compared with D, we find that the amplitude of the electric vector is proportional to

$$\sqrt{4\sin^2\frac{2\pi}{\lambda}D\cdot\sin\frac{2\pi}{\lambda}\frac{sx'}{D}+4\cos^2\frac{2\pi}{\lambda}D\cdot\sin^2\frac{2\pi}{\lambda}\frac{sx'}{D}}$$

and hence the total time average of the energy for all elements of the slit is proportional to

$$4\int_{d-\delta}^{d+\delta} \sin^2 \frac{2\pi}{\lambda}\frac{sx'}{D}\,ds = 4\delta - \frac{\lambda D}{\pi x'}\sin\frac{4\pi x'\delta}{D\lambda}\cos\frac{4\pi x' d}{D\lambda} \tag{5}$$

where δ is the half slit-width.

This clearly represents a beat phenomenon, the first factor of the second term being an amplitude factor, and the second giving the ordinary interference fringes. The change of x' from one maximum to another (the fringe width) is given by

$$\Delta x' \cdot \frac{4\pi d}{D\lambda} = 2\pi,$$

or

$$\Delta x' = \frac{D\lambda}{2d}; \tag{6}$$

while the positions of maximum visibility are separated by an amount given by

$$\Delta x' \cdot \frac{4\pi\delta}{D\lambda} = \pi,$$

or

$$\Delta x' = \frac{D\lambda}{4\delta}. \tag{7}$$

These two results, (6) and (7), were checked experimentally in the following manner: The width of the slit was measured by a micrometer comparator, and the observing microscope was placed on a micrometer mounting. The distance $2d$ was measured directly by focusing a real image of the slit and its image on the focal plane OO′ by means of a half-lens. Sodium light was used as a source. Distances between visibility minima were measured, since they are easier to observe than visibility maxima. In spite of all precautions, however, experimental data have an accuracy of only 4 to 5%. A typical set is given below:

D in mm	Slit Width $=2\delta$ in mm	Distance between Slits $=2d$ in mm	Fringe Width in mm		Distance between Visibility Minima in mm	
			Observed	Calc. $=\frac{D\lambda}{2d}$	Observed	Calc. $=\frac{D\lambda}{4\delta}$
723	.194	3.582	.118	.119	1.123	1.098
723	.241	3.688	.112	.115	.861	.882
723	.112	3.264	.130	.130	1.840	1.900

The agreement between observed and computed values is seen to be within experimental error.

An interesting point in this connection is the fact that (5) represents the same expression as would result from a replacement of the full slit by two non-coherent slits of infinitesimal width at the edges of the original slit. This was verified by placing a wire over the slit so as to cover all but a very narrow slit at each edge. The positions of the maxima and minima were undisturbed.

In conclusion, the writer wishes to express his thanks to Professor L. R. Ingersoll for suggesting the problem, and for his assistance in taking observations.

UNIVERSITY OF WISCONSIN,
DECEMBER 8, 1922.

OPTICAL SOCIETY OF AMERICA

NOTICE OF ELECTION OF OFFICERS

The following officers of the Optical Society of America will be elected by mail ballot in October, 1923, each to serve a two-year term beginning January 1st, 1924:

President,
Vice-President,
Four members of the Executive Council.

There will be no election of Treasurer nor Secretary this year.

The present officers are:

President: L. T. Troland
Vice-President: H. E. Ives
Secretary: Irwin G. Priest
Treasurer: Adolph Lomb.
Elected Members of the Council:
Adelbert Ames, Jr.
H. G. Gale
Ernest Merritt
W. E. Forsythe
Members of the Council *ex officio:*
The Past President: J. P. C. Southall
The Editor of the Journal: P. D. Foote
The Assistant Editor: F. K. Richtmyer

Neither the President nor Vice-President is eligible to re-election to the same office. W. E. Forsythe is not eligible to re-election to the Council. The other members of the Council are eligible to re-election.

The retiring President (Troland) will be *ex officio* a member of the Council, 1924–1925.

Pursuant to the By-Laws, the following committee on Nominations for these offices has been appointed by the President and approved by the Council:

J. P. C. Southall, *Chairman*
Columbia University, New York.
P. G. Nutting,
No. 4 Union St., Schenectady, N. Y.
Hermann Kellner,
Bausch & Lomb, Rochester, N. Y.
F. K. Richtmyer,
Cornell University, Ithaca, N. Y.
E. C. Crittenden,
Bureau of Standards, Washington, D. C.

The Secretary has requested the Committee to file its nominations with him not later than September 1st, 1923.

IRWIN G. PRIEST,
Secretary.

INSTRUMENT SECTION

A SPECIAL SENSITOMETER FOR THE STUDY OF THE PHOTOGRAPHIC RECIPROCITY LAW*

By Loyd A. Jones

In sensitometric research on photographic materials, it is necessary to expose samples of the materials being studied to radiation of precisely known intensity and quality for known periods of time. This is usually accomplished in an instrument commonly referred to as a sensitometer.

The product of exposure time (t) and the intensity of radiation (I) incident upon the sensitive surfaces is termed the exposure (E). It is evident that the exposure may be varied by either of the factors I or t. Sensitometers may therefore be divided into two general classes:

(a) *Time scale instruments:* In these the intensity incident upon the material is the same at all points, the desired variation in exposure being obtained by variations in the time of exposure.

(b) *Intensity scale instruments:* In these the time of exposure is constant and the intensity variable.

It was found by Hurter and Driffield[1] that if density be plotted as a function of the logarithm of the exposure, the resultant curve was a straight line over a considerable portion of its length. It has therefore been customary to construct sensitometers so that exposures from step to step on the material increase in geometrical ratio, thus giving points separated by equal intervals on the Log. exposure axis.

Almost all conceivable methods of varying the intensity of light have been employed in some form or other in the construction of sensitometers, and the literature of photography contains the description of many different types of such instruments. A fairly complete bibliography of sensitometric instruments will be found in a paper by Sheppard and Mees[2] and in a paper by the author[3] the subject is treated in a rather general way and a description of several types of instruments given.

* Communication No. 161 from the Research Laboratory of the Eastman Kodak Company.

[1] Hurter and Driffield, Phot. Jour., *14*, p. 217, 1890.

[2] Sheppard and Mees, Phot. Jour., *28*, p. 200, 1904.

[3] Jones, L. A., Phot. Jour., *40*, p. 80, 1920.

The choice of an instrument for use in a given problem depends very much upon the conditions and for special problems it is frequently necessary to construct sensitometers of special design.

The essential requirements of a sensitometer for investigation of the recriprocity law may be stated as follows:

(a) It must be of the non-intermittent type.

(b) The intensity of radiation (I) incident upon the sensitive material must be variable over a wide range of values.

(c) The method of varying the intensity must be such as to permit of a precise determination of the intensity values.

(d) The variation of intensity must be such as to preclude the possibility of any accompanying change in the quality (spectral distribution of energy) of the radiation.

(e) The exposure time (t) must be variable over a wide range of values.

(f) The method of varying the exposure time must be such as to permit of a precise determination of t values.

It has been definitely determined by Abney[4] and many succeeding workers[5] that an intermittent exposure is not integrated by the photographic plate. This immediately eliminates the possibility of using a sensitometer employing a rotating sectored wheel of the ordinary type. In such instruments, the photographic plate is subjected to an intermittent action of the exposing radiation and the blackening obtained is dependent to a great extent upon the ratio of the duration of each flash to that of the rest period between succeeding flashes.

As regards methods of varying the intensity, many of those in common use for visual work must be discarded on account of a lack of precise non-selectivity from the photographic standpoint. For instance, no filters precisely non-selective and having known transmission values are available. Reflecting surfaces in almost all cases show some selective effects. Nicol prisms are frequently selective in transmission and moreover the range available is not sufficient. The variation of illumination by changing the distance between the light source and the photographic plate is about the only one which is absolutely satisfactory. However, this method alone is not completely satis-

[4] Abney, Phot. Jour., *18*, pp. 56, 71, 1893.

[5] Schwarzschild, K., Astro. Phys. Jour., *11*, p. 92, 1900.
Howe, H. E., Phys. Rev., *8*, p. 674, 1916.
Englisch, E. W., Das Schwarzungs-gesetz. (W. Knapp, Halle.)
Sheppard and Mees, Investigations on the Theory of the Photographic Process. Longmans & Co., p. 222.

factory since it is impossible without the availability of very long distances to obtain the desired variation in intensity. Since the photographic material to be exposed must have sufficient area to accommodate the desired number of exposure steps, the minimum distance which is safe to use and assume the validity of the inverse square law is approximately 1 meter. In an investigation of the reciprocity law, it is desired to vary the intensity from 1 to a hundred thousand or even from 1 to a million. A photometric bar 1000 meters long would be required to obtain the latter range and it is quite evident that this is prohibitive.

Another method which can be used with safety is the variation of the effective size of the light source. Thus, if we use as a light source a surface having a known area, the effective illumination on the working plane can be varied by means of opaque diaphragms which limit the area of the source. Great care, however, must be used in employing this method and in the following description the factors requiring special attention will be emphasized.

After a careful consideration of the requirements of the problems and a survey of the various methods available for obtaining a satisfactory variation of the time and intensity factors, an instrument was designed and constructed. This embodies the principles which in the opinion of the author are most reliable and conveniently applicable to the problem in hand. A description of the essential structural details is given in the following pages.

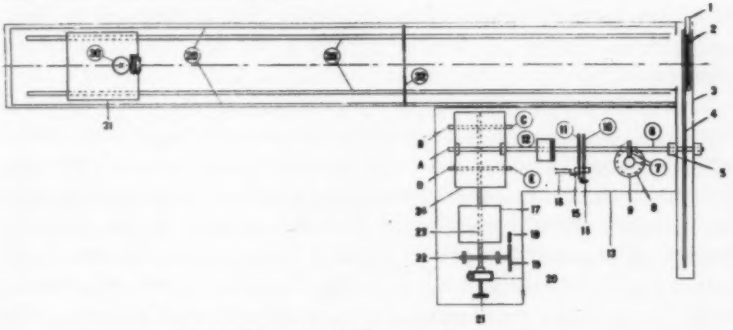


FIG. 1. *Plan view showing assembly of parts.*

In Fig. 1 is shown a general assembly view of the equipment as it was finally constructed. The light source (30) is mounted inside a light-tight lamp house (31). This lamp housing moves on a track (28) made by mounting one-half inch steel rods on suitable supports. This

track forms a photometric bar approximately 11 meters long, being of such length that the lamp may be set at any distance from 1 to 10 meters from the photographic plate (2). The track together with the lamp house is enclosed in a light-tight housing (20). In the front are numerous doors so that the lamp may be set in the desired position. Stray light arising from the internal reflections from the walls of the enclosing box is eliminated by suitably placed diaphragms. One of these (32) is shown in position. These diaphragms slide into the grooves cut on the inside of the housing. Grooves are provided at each half meter point and a set of diaphragms having apertures of various sizes is provided. With the lamp set in any position the diaphragms are adjusted to eliminate all possibility of reflected light reaching the photographic plate.

The timing mechanism consists essentially of a circular metal disk from which portions have been cut away. This is very similar to the sector wheels used in other forms of sensitometers. However, in this instrument an exposure is made by a single rotation of the sector wheel so that the errors arising from intermittency do not occur. This sector wheel (4) is mounted as shown on a 5/8 inch steel shaft carried in ball bearings (5). The photographic plate (2) is carried in the plate-holder (1) which slides into the position behind the sector wheel. The distances between the sector wheel and the surface of the photographic plate is reduced to the minimum (approximately 1/8 inch). The wheel, plate-holder, etc., are enclosed in a light-tight metal housing (3). The shaft (6) which carries the sector wheel is extended as shown and carries various appliances necessary for the operation of the instrument.

The wheel is driven by an 1/8 hp dc motor (17). The speed of the motor is controlled by a governor of the centrifugal type (20) so adjusted that the motor shaft (23) revolves at the rate of 1200 rpm. The motor shaft is extended through the gear box (24) in and on which are mounted the necessary gears for suitably reducing the speed for driving the sector. In order to obtain a wide variation in the angular velocity of the sector and hence a wide variation in exposure time, a rather complicated gear assembly is required. Most of these gears are assembled inside the gear box (24) which is filled with oil thus insuring smoothness of operation and a minimization of wear. Some of the shafts used in the gear mechanism projects through the sides of the gear box and at these points suitable spur gears may be attached so as to give the variable rotational velocities required. A detailed description of this gear mechanism will be given later.

The main shaft carrying the sector wheel articulates with the main shaft of the gear box through the magnetic clutch (12). It is necessary that some flexible connection such as this be included in order to avoid damage to the mechanism when the current through the motor is broken. Since the gear reduction mechanism includes worm and worm wheels elements which are not reversible, the inertia of the sector wheel when moving at relatively high angular velocities would tend to drive backwards through the motor when the motor circuit is broken and in this way damage to the gearing would result. The electro-magnetic clutch is so arranged, relative to the motor circuit, that no current can flow through it unless the motor circuit is closed. Moreover, the switch closing the clutch circuit is the magnetic type and opens automatically when the motor circuit is broken.

Since it is desired to expose the plate by a single revolution of the sector wheel, it is necessary to provide a means of turning on the light at the proper instant and cutting it off again after the wheel has made a complete revolution. This is accomplished by the cam plates (10) and (11) mounted on the main shaft together with the operating member (14) and the automatic circuit closing devices (15) and (16). The cam plate (10) is cut in such a way that the electric circuit can be closed only at the proper instant in the revolution of the sector wheel. Cam plate (11) is so cut that after the wheel has revolved through the desired angle, the member (15) is acted upon thus opening the switch (16). The switch (16) is in the circuit which operates the shutter in front of the lamp house. Details of this construction will be referred to later.

In a system in which a train of gears is employed, there is necessarily a certain amount of backlash. In order to hold this always in the same direction a brake is used. Braking action is obtained by rotating a metal disk in a tank of viscous oil. The pair of helical gears (7) are attached as shown, one member of which drives the brake disk (8) in the oil tank (9). In this way, a perfectly uniform load is applied directly to the sector wheel shaft and great smoothness of operation is obtained.

In order to check the speed at which the motor is operated a timing mechanism is attached. By means of a worm mounted on the motor shaft and a 60-tooth worm wheel, the disk (19) is made to revolve at 1 rps when the motor is operated at 1200 rpm. The disk (19) is of insulating material and on its periphery carries a contact point which as it passes under the member (18) closes a circuit through a relay and

sounder. A sounder attached to a standard clock beats seconds. The member (18) is mounted on an arm which may be rotated concentrically with shaft (22) so that the point of contact can be adjusted to any angular position. In this way the period of the two sounders can be brought into step and the centrifugal governor adjusted until synchronism is obtained. While the centrifugal governor has been found to be fairly satisfactory it does not function with the precision necessary to obtain precise synchronism over long periods of time. In order to correct this two switches so arranged as to increase or decrease slightly the resistance in the governing circuit are mounted on the control board. The operator can therefore pull the motor back into synchronism with the clock by closing one of these switches without in any interfering with the adjustment of the governor which on the average may be correct. The various elements are mounted on a cast iron L-shaped base (13).

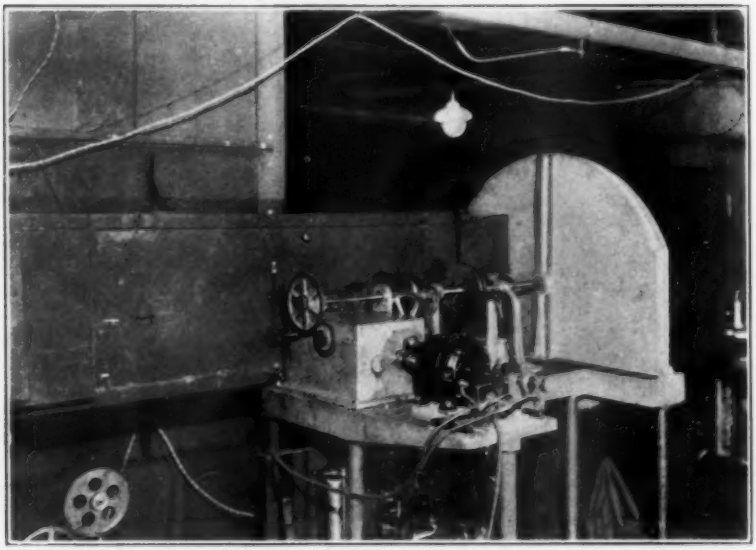


FIG. 2. *Photograph showing complete instrument.*

In Fig. 2 and 3 are shown photographs of the assembled instrument. In Fig. 3 a part of the sector wheel casing is removed showing the sector wheel in position. In Fig. 4 is shown the control board with relay, sounders, speed correcting keys, and magnetit clutch starting box in position. In Fig. 5 is shown a close up of the instrument. From left

to right the elements visible are motor with governor and speed indicating disk, gear box, electro-magnetic clutch, light control cams, viscosity brake and sector housing.

In Fig. 6 is shown the sector wheel, the shaded areas representing these portions which are cut away. A 30-inch steel saw blade about

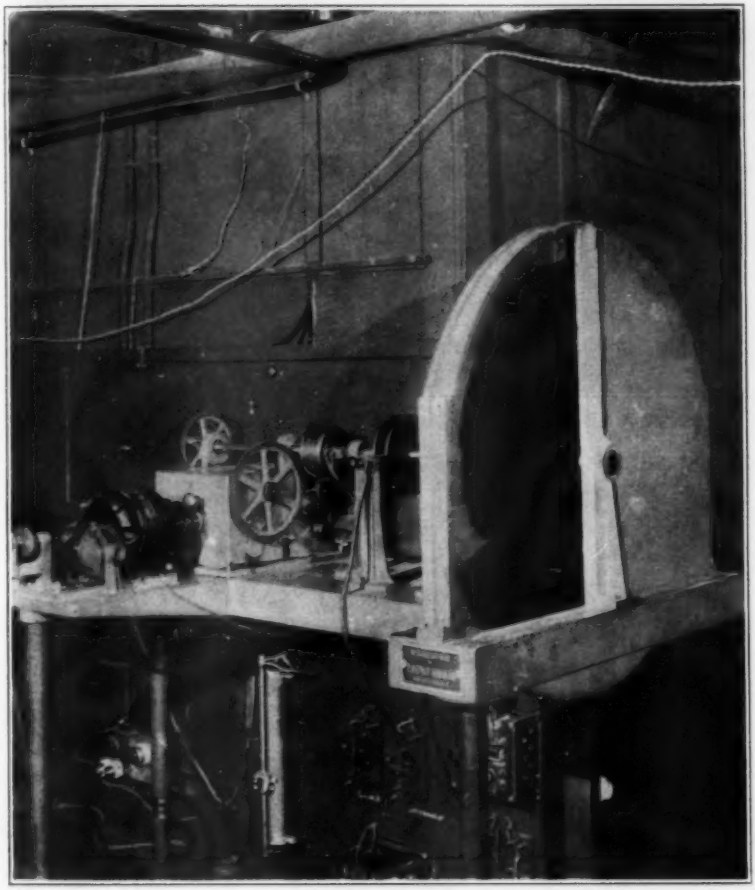


FIG. 3. *Photograph showing sector wheel casing removed.*

1/8 inch thick was used in constructing this disk. The maximum angular opening is 240° each succeeding step being one-half the preceding in angular dimensions. Ten steps are used, the minimum angular dimension being .469°. This is equivalent to an aperture .092

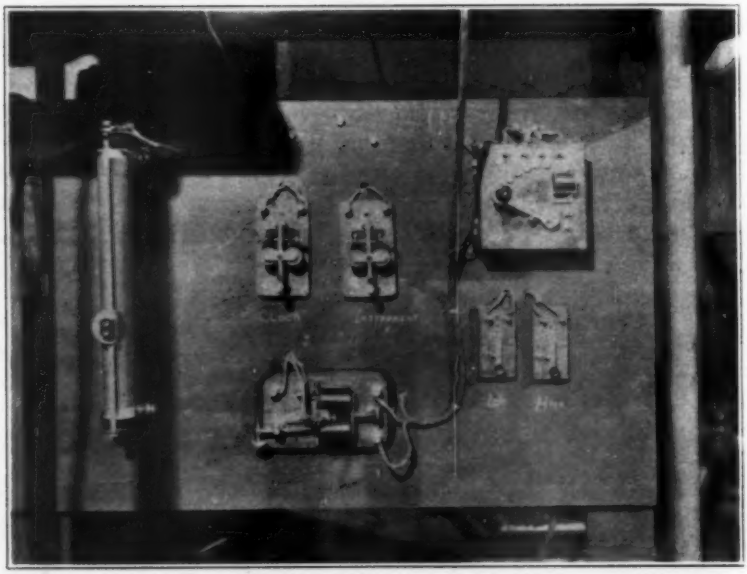


FIG. 4. *Photograph of control board.*

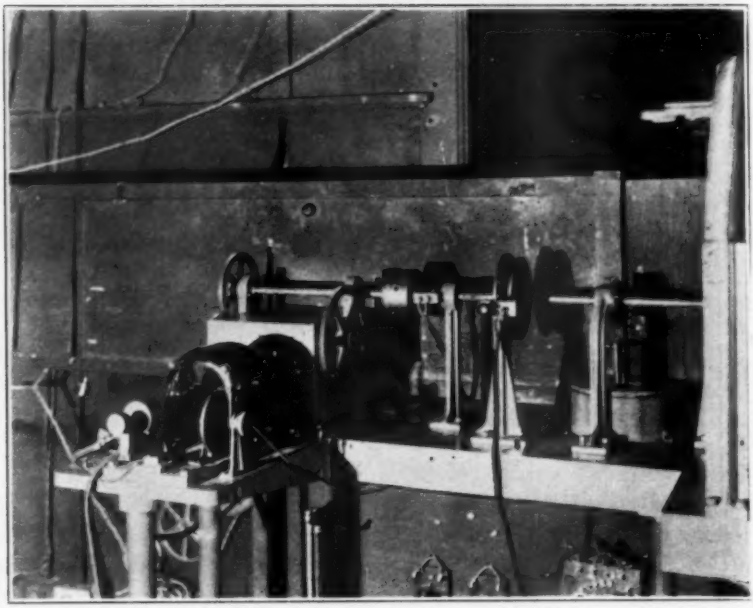


FIG. 5. *Photograph showing details.*

inch in width and is cut to a precision of .001 inch. The maximum error in exposure therefore resulting from errors in the cutting of the disk is of the order of 1%. In the case of the apertures of greater angular dimensions, the error is obviously very much less than this value.

In Fig. 7 is shown a detailed view of the gear box. This gear assembly permits an enormous variation in the angular velocity of the main shaft (4) and a more or less detailed description of the way it is constructed may be of interest.

The motor shaft is directly connected to shaft (1) through the flanged plate (2). This shaft terminates in another flanged plate (9).

FIG. 6. *Drawing of sector wheel.*

A second shaft (13) is mounted with its axis coincident with that of (1). The end of shaft (13) carries the clutch plate (17). This clutch plate is free to slide back and forth on shaft (13) but when rotated carries the shaft with it by a sliding key which sets in a key-way cut in shaft (13). When this clutch plate is pulled toward the motor by means of a suitably mounted lever, the projections on its face engages with holes in the flanged plate (9) thus causing shafts (1) and (13) to function as a unit. When, however, the clutch plate is moved away from the motor projections on the opposite face engage in openings in the hub (10) of the spur gear (16). This spur gear is mounted on the shaft (13) but is free to rotate on that shaft except when in engagement with the clutch plate.

There are two positions for the clutch plate, one for high speeds and one for low speeds as indicated by the designations in the side elevation. When set for high speeds the drive of the main shaft carrying the sector wheel is through the counter-shaft (14) which carries the 60-tooth worm wheel (12) driven by the worm (11) on the shaft (13). Under such conditions the angular velocity of shaft (14) is 20 rpm. Counter-shaft (14) drives the main shaft through a pair of spur gears placed at A and B. Four pairs of spur gears are provided, any pair of which may be placed on A and B.

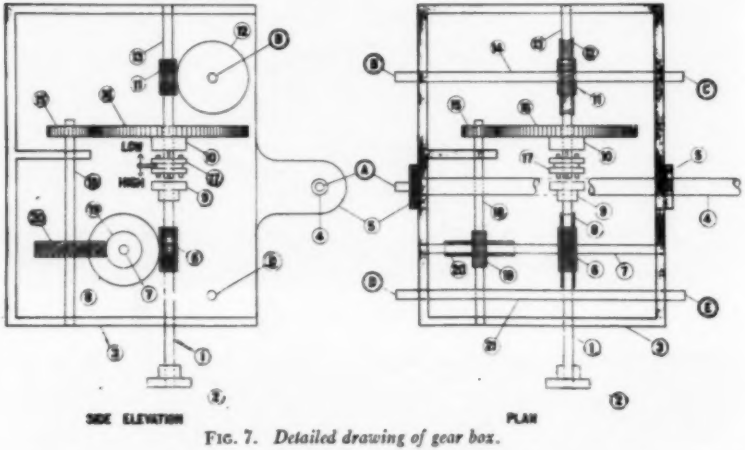


FIG. 7. *Detailed drawing of gear box.*

The gears used are as follows, the designation being by the number of teeth:

Pair No. 1 90–90
Pair No. 2 60–120
Pair No. 3 36–144
Pair No. 4 20–160

Rotational velocities obtainable when mounted at A and B are therefore:

A	B	Ratio
20	160	1 to 8
36	144	1 to 4
60	120	1 to 2
90	90	1 to 1
120	60	2 to 1
144	36	4 to 1
160	20	8 to 1

The idler shaft (21) is mounted as indicated with its ends D and E projecting outside the gear case. The centre line distance between (21) and (4) is equal to that between (14) and (4) so that articulation may be accomplished between A and D by the same pair of spur gears as are used at A and B. The idler shaft is driven by suitable spur gears located at C and E. In this case drive must be through an idler spur gear in order to obtain the proper direction of rotation. The distance between C and E is therefore somewhat greater than between D and E. Driving with the clutch plate in the position marked "high" rotational velocities from 1280 rpm to .305 rpm may be obtained by proper adjustment of the spur gears on the outside of the box. Now, when the clutch plate is moved to the position marked "low" drive is accomplished through worm (6) mounted on shaft (1) and the 64-tooth worm wheel (8) on counter-shaft (7) thence through a pair of helical gears (10) and (20) (ratio 1 to 2) thence through the pair of spur gears (15) and (16) (ratio 1 to 4). Clutch plate (17) now rotates with spur gear (17). This now gives a reduction of 512 and by proper adjustment of the spur gears on the outside of the box, rotational velocities of from 2.5 to .000614 rpm may be effected.

If we designate by t (max) the exposure time given to step No. 1 on the exposure strip, that is, the step exposed through the sector wheel slot having an angular opening of 240°, then t (max) may be varied from .03125 seconds up to 18 hours, 12 minutes and 16 seconds, a range of 1 to 2 000 000. It has been found impracticable to operate the sector wheel at anything greater than 320 rpm. The light operating mechanism has been found not to function satisfactorily for the higher velocities. However, exposures have been made with t (max) equal to .125 seconds with perfect satisfaction. (In Table No. 1 is shown the adjustments giving the various t (max) values.) This gives a range in t (max) from 1 to 524 288 and since there is a range of 1 to 512 due to the sector wheel itself, the total range from maximum T (max) to minimum t (min) is 1 to over 268 million.

This range of t values is sufficient to include the fastest focal plane exposures on the one hand and the longest astronomical exposures on the other, thus embracing the entire working range of photographic materials.

In Fig. 8 is shown the details of lamp house No. 1. The lamp F is mounted in a fixed position inside the housing. The solenoid A is connected to the switch (16) shown in Fig. 1. When the circuit is closed through this coil the shutter B is deflected allowing the light

from the lamp to illuminate the photographic plate. The solenoid system has been designed particularly so as to obtain a very high speed of operation and it is estimated that the aperture is completely opened within 1/100 of a second after the closing of the circuit. The index marked E is attached to the lamp house thus permitting the lamp to be set at the desired distance from the photographic plate, a scale

TABLE 1. *Data relative to gearing*

Gears					*Shift*	*P P M*			*t (max)*		
A	B	D	C	E	Lever	14	21	4	h.	m.	s.
20		160	160	20	High	20	160	1280			.03125
20		160	144	36	"	20	20	640			.0625
20		160	120	60	"	20	40	320			.125
20	160				"	20		160			.25
36	144				"	20		80			.5
60	120				"	20		40			1.0
90	90				"	20		20			2.0
120	60				"	20		10			4.
144	32				"	20		5			8.
160	20				"	20		2.5			16.
160		20	60	120	"	20	10	1.25			32.
160		20	36	144	"	20	5	.625		1	4.
160		20	20	160	"	20	2.5	.3125		2	8.
20		160	160	20	Low	.039	.625	2.5			16.
20		160	144	36	"	"	.312	1.25			32.
20		160	120	60	"	"	.156	.625		1	4.
20	160				"	"		.312		2	8.
36	144				"	"		.156		4	16.
60	120				"	"		.078		8	32.
90	90				"	"		.039		17	4.
120	60				"	"		.019		34	8.
144	32				"	"		.0097	1	8	16.
160	20				"	"		.0048	2	16	32.
160		20	60	120	"	"	.019	.0024	4	33	4.
160		20	36	144	"	"	.0097	.0012	9	6	8.
160		20	20	160	"	"	.0048	.0006	18	12	16.

being mounted in position for that purpose. At C are three plates of white optical glass about 3 mm thick. Further reference will be made to this later. A photograph of this lamp house with the solenoids which operate the shutter is shown in Fig. 9.

In Fig. 10 is shown details of a second lamp house (No. 2) used when low illuminations are desired. In this case the electro-magnetically

operated shutter is dispensed with, the lamp itself being directly connected to the timing switch (16) shown in Fig. 1. When low illuminations are used the rotational velocity of the sector wheel is so small

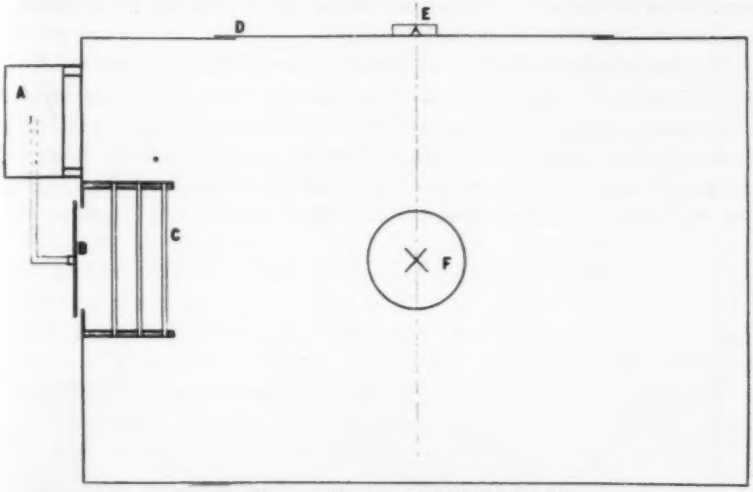


FIG. 8. *Diagram of lamp house for high intensities.*

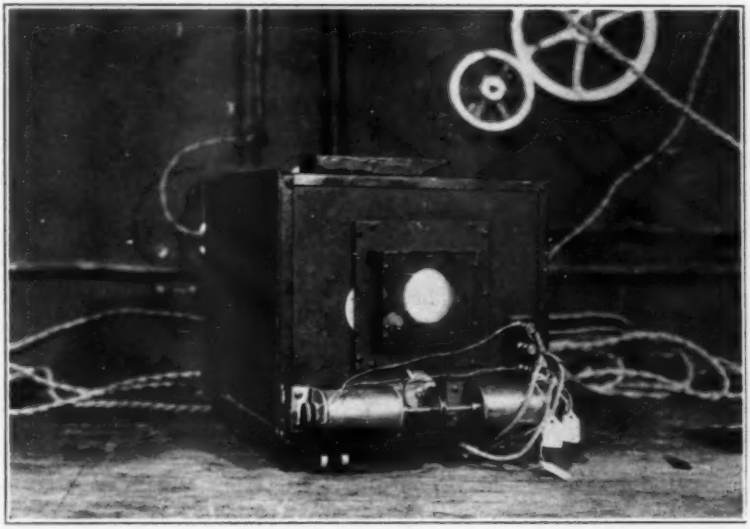


FIG. 9. *Photograph of lamp house showing electro-magnetic shutter.*

that there is sufficient time for the lamp to be turned on, come up to voltage and be checked by the operator before the actual exposure begins. This is not true in the case of the higher intensities and high rotational velocities of the sector wheel and hence the magnetically operated shutter was attached.

Turning again to Fig. 10, at A are mounted three plates of white optical glass exactly similar to those used in the other lamp house. This exact similarity was insured by cutting one plate for each lamp house from the same sheet of glass. The glass used in this case however is fine ground on both sides. The separation between elements is about 1 cm. The front face of this diffusing unit now becomes the

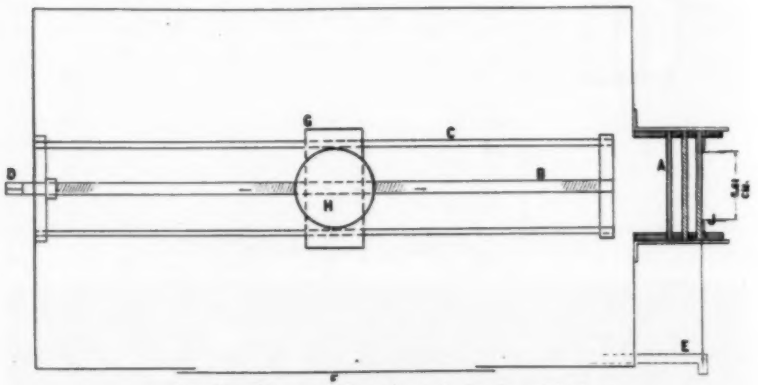


FIG. 10. *Diagram of lamp house for low intensities.*

effective light source and the area is limited by the opaque diaphragm J placed in contact with the front surface of the diffusing assembly. The great care taken in having exactly the same glass in both lamp houses is to avoid the possibility of any differences in the quality of the light in the two cases. The lamp H is mounted on a fixture carried by a sliding block G which is mounted on the track C. The position of this lamp can be adjusted very precisely by a rotation of a helical screw B. A set of diaphragms is provided for this lamp house; the largest one shown in position in Fig. 1 has an opening area of 100 sq. cm. The other members of the set are so cut that the area is decreased by one-half for each succeeding member. In this way it is possible to limit the effective area of the light source to 100, 50, 25, etc. down to .78 sq. cms.

If the diffusing element were known to be perfectly uniform and completely diffusing it would be permissible to compute the relative candle power with each of the various diaphragms by a precise determination of the areas. It was not considered satisfactory procedure, however, to make such an assumption and candle power measurements were made with great care using the various diaphragms. Small departures from proportionality to area were found and the actually measured values are used in preference to those depending upon area.

An index E is attached to the lamp house on a line coinciding with the outside surface of the diffusing element. This is used for setting the lamp at the desired distance from the photographic plate. A 100 watt lamp operated at 2500°K in this house was adjusted in position so that the effective candle power with the largest aperture in position gave an effective candle power of 1.0. Lamps of 100, 250, and 500 watt size carefully color matched on the bench photometer were used in lamp house No. 1 (Fig. 3). With this combination of lamps and housings, it was possible to obtain on the photographic plates, illuminations varying from 131.2 meter candles down to .000687 meter candles, a range of 1 to 200 000 and if necessary, this can easily be extended to 1 to 1 000 000.

The lamps used are mounted in a fixture which fits inside the lamp house. This fixture is so constructed that it may be removed from the lamp house and returned to exactly the same position. In this way lamps can be photometered in position inside the lamp house and in case necessity arises may be removed and replaced with a certainty that they have returned to the position occupied through the measurement.

Extreme care has been exercised throughout to provide the highest possible precision in the determination of the illumination incident upon the plate and it is thought that by proper use of the equipment, an illumination value can be determined to well within $\pm$ 1%. Likewise, the exposure times are reliable to within about the same limits.

The instrument has been in almost continuous use for the past six months and seems to be functioning satisfactorily. Results obtained will be presented in a later paper.

EASTMAN KODAK CO.,
ROCHESTER, NEW YORK.

The Quantum Theory. By Fritz Reiche. Translated by H. S. Hatfield and Henry L. Brose. 183 pages. E. P. Dutton & Co., 681 5th Ave., New York.

The author gives a birds-eye view, in the 126 pages of text, of the origin and development of the Quantum Theory, and its present ramifications and applications. Chapters I and II are devoted to a discussion of the failure of classical laws to give a satisfactory radiation formula, and a brief outline of Planck's development in which the Quantum Hypothesis was first proposed. Then follows (Chap. III) an account of the extension of the Hypothesis, by Einstein and others, to photoelectric and kindred phenomena. In Chapters IV and V are given the application of the Quantum Theory to molecular phenomena, such as specific heats, and to the electron theory of thermal and of electrical conduction. The remainder of the text (Chaps. VI to IX) is devoted to the application of the Quantum Theory to Spectral Phenomena.

By omitting from the text a large part of the mathematical discussions, and by giving in most cases only the end results the author is enabled to make the physical principles involved stand out the more clearly.

The 50 pages of references and notes constitute a very valuable feature. The book is suitable either for the reader who wishes a concise account of this fascinating field, or for the student, who, by aid of the copious references to original sources, wishes to go more deeply into the subject.

F. K. RICHTMYER

NOTE ON BRÄUER'S THEORY OF THE FUNCTION OF THE CATHODE IN ELECTRIC ARCS

By A. Trowbridge and K. T. Compton

In a recent paper in the Annalen der Physik, Ernest Bräuer[1] has described experiments which, he believes, disprove J. J. Thomson's theory that the principal part of the current in an electric arc is the thermionic current emitted from the hot cathode. Bräuer then proceeds to build up a new theory of the arc to accord with his interpretation of his experiments. We wish, in this note, to describe experiments which verify his experiments but disprove his interpretation of them.

Bräuer suddenly short circuited an arc with a resistance of such value as to leave a potential difference of a few volts, insufficient to ionize the gas, between the arc terminals. An oscillograph introduced in series with the arc indicated a small current, amounting to a few percent of the original arc current, persisting after the reduction in voltage and quickly falling to zero as the electrodes became cool. Bräuer interpreted this "post-arc" current as the measure of the thermionic current from the hot cathode, and considered it to represent the saturation value of the thermionic current because it was not appreciably increased by increasing the applied voltage from 10 to 20 volts.

But Bräuer failed to consider the effect of space charge in limiting the thermionic current[2] to a small value, however hot the cathode may be. Calculation shows that the maximum possible thermionic currents under the conditions of Bräuer's experiments could not have exceeded $0.1\ (10)^{-3}$ amp. per cm^2, which is less than 0.001 of the current which he measured.[3] He did not, therefore, measure the saturation thermionic current, and hence his argument is invalid.

In seeking to account for his experimental results it occurred to us that the only way to secure currents larger than those determined by the space charge around the cathode is to neutralize part of this space charge by positive ions, thus permitting the escape of additional electrons from the cathode. Since there was no ionization of the gas between the electrodes in Bräuer's experiments, these positive ions must have come from the anode. An estimate of the amount of positive

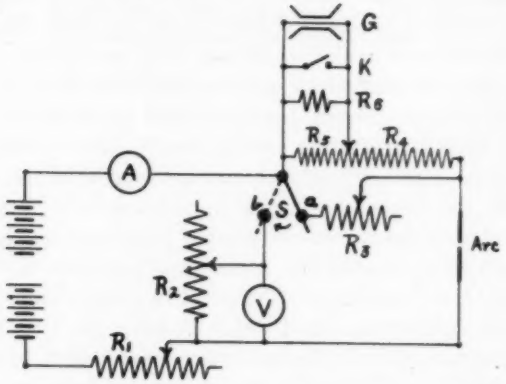
[1] Ann. d. Phys. *60*, p. 95; 1919.

[2] Langmuir, Phys. Rev., *2*, p. 457, 1913; Richardson, Phil. Mag. *32*, p. 426, 1916.

[3] K. T. Compton, Phys. Rev., (in press).

emission from the hot anode, together with the fact that each positive ion permits the escape of many electrons from the cathode because of its relatively slow motion through the region of the negative space charge, indicates that such action seems adequate to account for Bräuer's observations.

This suggestion was easily tested by varying the temperature or rate of cooling of the anode, and measuring the "post-arc" currents under these conditions. If the post arc current is, as Bräuer believed, the saturation thermionic current from the cathode, it should be independent of these changes in the anode temperature. If, on the other hand, our suggestion that Bräuer really measured the current of positive ions emitted by the anode (magnified by the negative current which these allow to escape from the cathode) is correct, then the large



post-arc current should be absent in the case of a cold anode and should decrease rapidly with time in the case of a rapidly cooled anode. The following experiments prove the correctness of this suggestion.

We employed essentially the method used by Bräuer, but modified so as to permit both the arc and the post-arc currents to be recorded by the oscillograph. As an oscillograph we used the string galvanometer G and photographic recording apparatus which has already been described by one of us.[4] The diagram of connections is shown in Fig. 1. It will be observed that, with the switch S thrown to position a, the galvanometer system is shunted across a low resistance R_3, whereas when the switch is thrown to position b, short-circuiting the arc through

[4] A. Trowbridge, Frank. Inst. Jour., Dec., 1922.

resistance R_2, the shunted galvanometer system is connected directly in series with the arc. The sensitivity of the galvanometer system is therefore less for the arc current than for the post-arc current. By adjusting R_3 it was possible to get practically the same deflection for the arc as for the initial post-arc currents, so that the ratio of the currents in the two cases was the inverse ratio of the corresponding sensitivities. R_1 was adjusted to give a convenient arc current and R_2 was adjusted to give a potential drop of 12 volts across the arc after the switch S was thrown from a to b. Resistances R_4, R_5, and R_6 were chosen to give proper damping and convenient sensitivity to the galvanometer system. The values chosen for these resistances were $R_3 = 1.67$, $R_4 = 275$, $R_5 = 60$, $R_6 = 0.5$ ohms. These resistances made the galvanometer 0.0077 times as sensitive for the post-arc as for the arc current.

Fig. 2 (a) illustrates the nature and interpretation of the photographic records of the galvanometer deflections. The vertical lines mark time intervals of 0.01 second. Up to the point A the switch S was at a, but the galvanometer was short-circuited by the key K to give zero deflection. At A the key K was opened, and the deflection d_1, measured the arc current i_1. At B the switch S was thrown to b, and the deflection d_2 measured the initial post arc current i_2. This current fell to zero at C in a few hundredths of a second, as the electrodes cooled. Owing to the difference in sensitivity in the two cases,

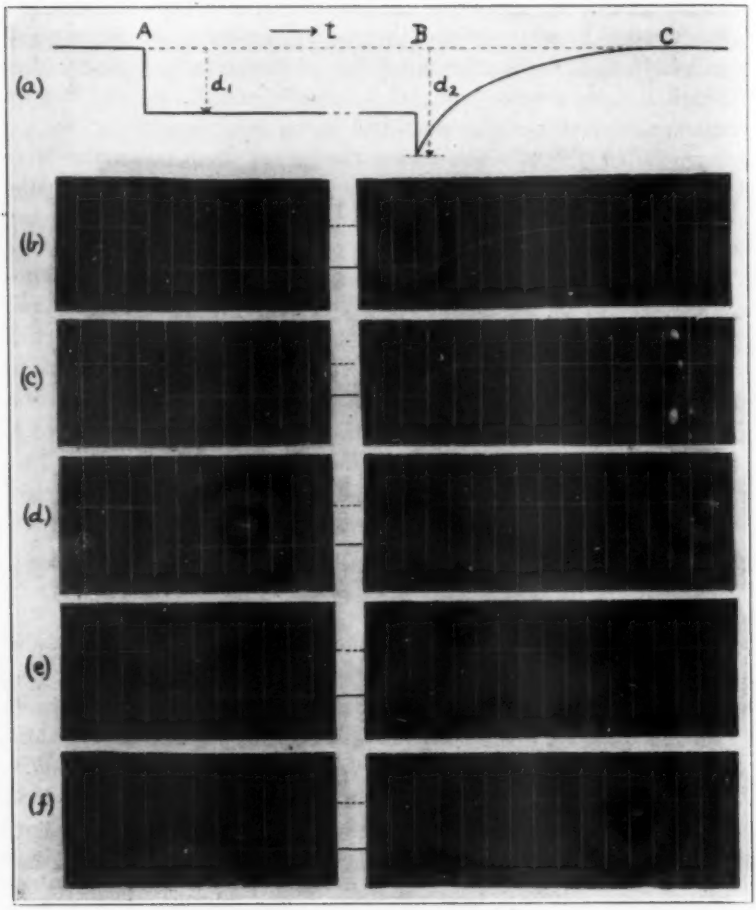
$$\frac{i_2}{i_1} = 0.0077 \frac{d_2}{d_1}.$$

In Fig. 2 are also shown reproductions of some of the actual results under various conditions. In 2 (b) two uncored carbon electrodes were used. The arc current was $i_1 = 3.7$ amps. and the post-arc current was found to be 0.034 amp. Thus $i_2/i_1 = 0.92$ per cent, which is close to the value found by Bräuer under similar conditions.

In 2 (c) the same electrodes were used, but a massive copper block was slipped over the anode at a distance of ½ inch from the hot point. This greatly increased the temperature gradient to the anode, causing it to cool more quickly. It is seen that the post-arc currents decreased with time much more rapidly than before.

In 2 (d) the copper block was 1/8 inch from the hot point. Here the cooling was so rapid that the post-arc current fell to zero almost instantly, in about 0.002 sec.

Intermediate stages could be obtained. The small vibrations shown at the tradition points of the curve are chiefly due to natural period of the string and possibly also to a small spark which occurred when the switch was thrown from *a* to *b*.



Curve 2 (e) shows the results when a water cooled copper anode was substituted for the carbon anode. Here there is no trace of a post-arc current.

Curve 2 (f) was taken to test the effect of impregnating the carbons with a sodium salt. As is to be expected, the arc current is increased, but the post-arc current is decreased since, in this case, the anode does not become hot as in the case of pure carbon electrodes.

These experiments prove conclusively that the positive ions emitted from the anode were responsible for the currents observed by Bräuer. Without their presence to break down the negative space charge, the thermionic emission from the cathode is limited to a much smaller value. The experiments therefore are in support of J. J. Thomson's view that, although the greater part of the arc current is due to thermionic emission of electrons, ionization is necessary to supply the positive ions required to neutralize the negative space charge and permit the electrons to escape from the cathode. Owing to the large suppressed emission from the cathode, the space charge remained negative in Bräuer's experiments, so that the positive emission from the anode reached its saturation value. This accounts for his observation that the currents were nearly independent of the post-arc voltage.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY.

Absorption Spectra of Thallium and Indium Vapors.—Whether or not there exists a $1s$ state for the metals of Group III of the periodic table has been for several years a puzzling question. In the empirical development of the arc series for aluminium, gallium, indium, and thallium this term does not appear. Although the spectra of these elements are doublets like the alkalis, the highest known convergence frequency is $2p_2$, whereas the highest term in the spectra of the alkalis is $1s$. Bohr's new theory of atomic structure indicates that the last added electron in the formation of each of these elements revolves in an orbit characterized by two quanta of angular momentum. This is in agreement with the idea that $2p$ represents the normal state of the atom and that there exists no $1s$ state. Accordingly the absorption spectra of these metals should be lines of the two subordinate series, not lines of the principal series as with the alkalis. The absorption of thallium and indium has been carefully investigated by Grotrian. From purely statistical considerations it follows, since the state $2p_2$ represents a more degraded system than the state $2p_1$, that more atoms will exist at low temperatures in the former condition. At low temperatures, accordingly, lines of the series $2p_2-ms$ and $2p_2-md$ should appear as reversals. As the temperature is raised however, the other terms of these doublets appear so that the series $2p_1-ms$ and $2p_1-md$ show absorption. The difference in temperature required to bring out the two members of these doublets should be greater the larger the difference in energy levels corresponding to $2p_2-2p_1$. All of these predictions were verified precisely. At 400 to 600°C thallium vapor showed only the absorption of the lines $2p_2-ms$ and $2p_2-md$. Three lines in each series were observed, the higher terms lying too far in the ultraviolet to be detected by the quartz spectrograph. At temperatures from 800 to 1200°C absorption of the lines $2p_1-ms$ and $2p_1-md$ appeared. The latter series was observed to the term m = 17!

Similar results were obtained with indium. Since the doublet separation $2p_2-2p_1$ for indium is only $\frac{1}{3}$ that for thallium, the temperature interval between the excitation of the p_2 terms alone and both numbers was found to be only 100 to 150° whereas an interval of 400° was necessary for thallium.

This work practically proves that the ionization potentials of the elements of this family should correspond to the wave number $2p_2$. It again emphasizes the importance of repeating the determinations of the critical potentials of thallium and of devising some method for measurement of the critical potentials of the more refractory elements of this group. [Walter Grotrian, Zeit. f. Physik *12*, pp. 218-31, 1922.]

PAUL D. FOOTE

A NEW FORM OF PRECISION HYDROMETER[1]

By C. W. Foulk

This paper gives a preliminary discussion of a new form of apparatus for the measurement of densities of liquids. The principle employed is that of submerged floating equilibrium and as it is described here the instrument can be constructed by any chemist or physicist and after proper calibration densities can be determined with it in a few minutes that will be accurate within two or three units in the fifth decimal place.

Active work by the author and his students is in progress to develop certain structural points of the apparatus so as to make it a serviceable industrial instrument, but since a year or more will necessarily be consumed in putting such a thing on the market, the present paper containing a discussion of its essential points is offered at this time.

Warrington[2] and more particularly, Richards[3] and his students have published a number of admirable papers on free floating equilibrium bobs. These are elongated hollow glass or silica bulbs so ballasted that they neither rise nor sink in a given liquid but remain submerged between the surface and the bottom. In other words, a bob in submerged floating equilibrium has the same density as the liquid. Such floats indicate density changes in the seventh decimal place but as measuring instruments they are afflicted with the limitation that a given bob can measure only one density. Their great sensitiveness and freedom from the errors of the ordinary hydrometer seemed, however, fairly to cry out for some modification that would give them as measuring instruments the distinguishing property which the projecting stem bestows upon the ordinary form of hydrometer, namely, the power automatically to adjust its weight to the density of the liquid in which it is placed. The author thought for about a year that he was the first to solve this problem but later discovered that it had been done in 1864 by Saxton[4] who attached a chain to the float as shown in Fig. 1.

[1] Application for patents covering all forms of this device have been made and arrangements have been entered into with Kauffman Lattimer and Company of Columbus, Ohio to manufacture and sell a commercial instrument when it is ready.

[2] Phil. Mag. *48*, p. 498; 1899.

[3] Richards and Shipley, J. Am. Chem. Soc. *34*, p. 599; 1912 and *36*, p. 1; 1914. Also Richards and Harris, ibid. *38*, p. 1000; 1916.

[4] U. S. Patent 44460 (1864).

Petterssen[5] in 1917, apparently in ignorance of Saxton's work, again arrived at the same idea.

Neither investigator, however, hit upon the plan of hanging the chain in a catenary curve between the float and an independent point of support as is shown in Fig. 2. The instrument there illustrated consists of a tube open at both ends and of such diameter that the float can move freely along its length. An appropriate scale is etched on the side of this tube and by means of an arbitrary mark the position of the float on the scale is determined.

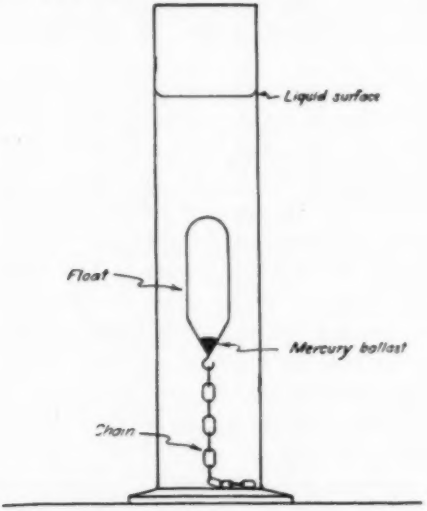


FIG. 1

When this apparatus is put into a liquid the float at once takes up a position which is determined by the density of the liquid, because the suspended chain automatically adjusts its weight to that density. If the first position of the float is too high it will, of course, sink, but in doing so the part of the chain hanging from it becomes shorter and consequently lighter because the weight is transferred to the hook on the bottom of the tube. The reverse process takes place if the float happens to be too low at first.

On comparing this form with that shown in Fig. 1 several important advantages can be seen in favor of the catenary hanging of the chain. (1) The sensitiveness of the float to changes in density in the liquid is

[5] U. S. Monthly Weather Review *45*, p. 159 and p. 236; 1917.

just double that of the same float and chain when used as in Fig. 1. This means that an increase of density in the liquid that would cause the float to rise 1mm if the chain rested on the bottom of the vessel would bring about a rise of 2 mm if the chain hung in a catenary. (2) Changes of density of the liquid, however small, bring about proportional changes in the position of the float. This is not true of the arrangement of Fig. 1. Here the float in response to a gradual density change of the liquid will sink or rise, in stepwise fashion, according to the length of the links. (3) The catenary hanging of the chain

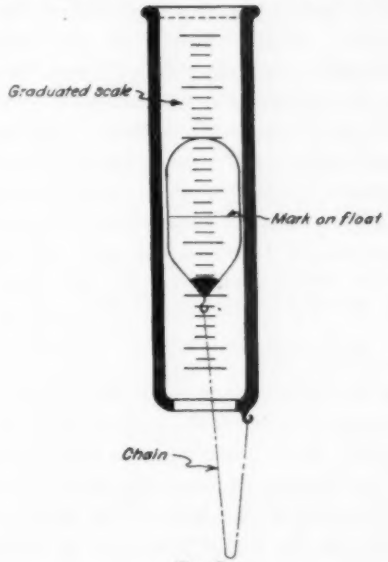


FIG. 2

prevents tangling and piling of links on each other as might be the case if the supported part rested on the bottom of the vessel. (4) The form of apparatus as shown in Fig. 2 is more easily handled than a float and chain alone would be.

Points (1) and (2) have been checked experimentally and found to hold true as closely as the instrument can be read.

The effect of the various constants of such a balanced float on its response to differences of density of the liquid in which it is placed is given with close approximation by the following equation[6]:

[6] From an unpublished paper on the mathematics of chain balanced floats by C. W. FOULK and D. B. BROOKS.

$$\Delta d = \frac{(D-d)\times \Delta b}{2\,D\times V} \times w$$

Δd = density change in liquid which causes a vertical displacement of float of Δb measured in cm.

D = density of material of chain.

d = density of liquid.

V = volume of float measured in cc.

w = weight in grams of 1 cm of chain.

Carefully conducted experiments with liquids whose densities were determined with an Ostwald-Sprengel pycnometer indicated that this formula was accurate within about 3%. The experimental values showed a greater sensitiveness than the calculated ones.

By way of practical illustration the values for an actual instrument will be given. A "gold" (note the density!) eyeglass chain weighing 0.0145 g/cm and density 10.7 was used with an 8 cc float in a liquid with a density approximately 1. Now, assuming that it is desired to know what change in density of the liquid will bring about a vertical displacement of 1 mm of the float, the equation becomes,

$$\Delta d = \frac{(10.7-1)\times 0.1}{2\,(10.7)\times 8} \times 0.0145$$

$$\Delta d = 0.0000833$$

This means that a density change of 8 in the fifth decimal place causes a movement of 1 mm by the float, and since on a millimeter scale readings can be made with no greater error than 0.2 to 0.3 mm it can be said of such an instrument that the density measurements are defined within 2 in the fifth place. This presupposes, of course, accurate calibration of the instrument, care in using it, and proper temperature control. Such things are true, however, of all precise measurements.

An inspection of the formula shows that the sensitiveness of the instrument is proportional to the volume of the float and inversely proportional to the weight of unit length of the chain. The effect of an increase in the density of the chain material is to diminish the sensitiveness by a small amount. If in the above practical illustration the density of the chain were doubled the sensitiveness of the instrument would change from 0.000083 to 0.000086 about 3.6%. Similarly the density of the liquid affects the sensitiveness by a small amount only and in the sense that increased density increases the sensitiveness. In the illustration above, doubling the density of the liquid would

increase the sensitiveness so that a density change of 0.000074 instead of 0.000083—a difference of about 10.8%—would cause the float to move 1 mm.

Irregularities of weight along the length of the chain would occasion errors but owing to the nature of their manufacture chains can be produced of great uniformity.

The smallest chain which has been found on the market is the gold eyeglass one referred to above. Another gold plated chain for hanging pencils around the neck was found which weighed about 30 mg to the cm. If this is used with a 12 cc float a millimeter change in position indicates a density change of 0.000113.

Increasing the size of the float is perhaps the easiest way of increasing the sensitiveness of the apparatus but if carried too far, serious objections would arise in its practical use. Decreasing the size of the chain is more practical because small floats can then be used. The author has at hand some fine platinum wire and has experimented with it enough to feel sure that a chain can be made that will weigh 1 mg to the cm.

Other schemes have been used for giving a submerged float a certain range as a density measuring instrument. Warrington,[7] working with free floating bobs weighted them by slipping rings of platinum wire over a stem at the top. By using an appropriate system of such weights, he could change the total weight of the float (within certain limits) by units of 1 mg. Finer adjustments of equilibrium had to be made by varying the temperature of the liquid.

Lamb and Lee[8] also weighted a submerged float but they used a simpler scheme. Platinum weights were laid on the flat top of the float. The final equilibrium was adjusted and measured by a magnetic pull exerted on a piece of iron sealed inside the float. This magnetic effect was produced by a current sent through a helix of copper wire. The current was measured in milliamperes. With a bob of 235 cc volume, an extraordinary degree of precision was obtained by this method. With their apparatus, Lamb and Lee obtained density readings that checked usually within a few units in the eighth decimal place and which they considered accurate within less than three units in the seventh place.

It is interesting to calculate the sensitiveness of a chain float of 235 cc volume and a platinum chain weighing 1 mg to the cm. A change

[7] Ref. 2.

[8] J. Am. Chem. Soc. *35* p. 1666; 1913.

in density of the liquid of 2 in the sixth decimal place would cause the float to move over 1 mm.

A striking feature of the balanced type of equilibrium float is the variety of modifications that can be given it. For example, if two non-miscible liquids are used with the bob in equilibrium in the upper layer and with a fine wire extending from it downwards into the lower and heavier liquid, a system is produced that is extremely sensitive to changes in density in the upper layer and yet possesses the automatic adjustment features of the chain bob. This is illustrated in Fig. 3. If the bob rises, a short length of the wire is lifted from the heavier

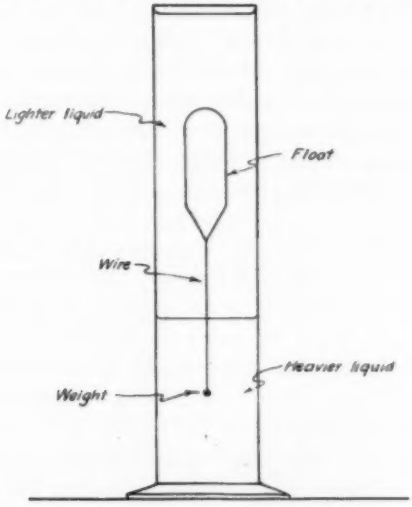


Fig. 3

into the lighter liquid, thereby making the floating system heavier and restoring the equilibrium. The sensitiveness is high because the change in net weight is measured not by the weight of wire lifted from one liquid to the other, but by the difference in weight of the two volumes of liquid displaced by that portion of the wire that moves from one layer into the other. By using a fine wire and two liquids of nearly the same density, it is evident that very high sensitiveness can be attained. Such an instrument would have little use in determining densities but might serve some other scientific purpose.

A third type which when developed may prove the most accurate as a measuring device is illustrated in Fig. 4. One end of the chain

is attached to a rod which can be moved up or down. The bob is first allowed to come to equilibrium and is then brought to an arbitrary mark on the side of the tube. The vertical distance through which the rod moves to bring the bob to this point will give the final essential data for obtaining the density of the liquid. The advantages of this form lie in the ease with which the vertical movement of the rod can be measured, by vernier or micrometer screw, and the accuracy with which the bob can be brought to a definite position.

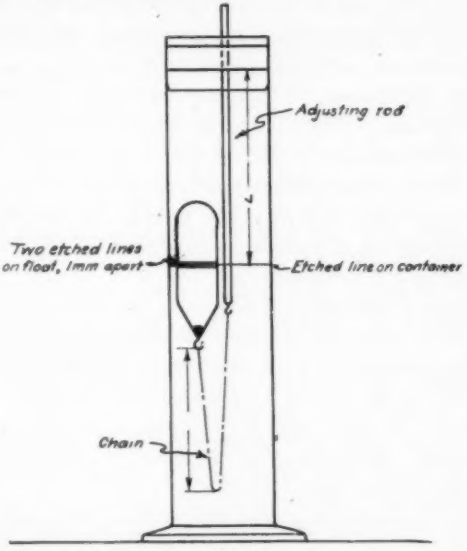


FIG. 4

Another interesting variation depends upon a device for increasing the length of a link of a chain without materially increasing its weight. This is easily accomplished by separating a chain into its individual links and then reconstructing it by tying the links together with very fine thread. Since cotton or silk thread has a low density, a few inches of it has little weight when submerged in an ordinary liquid and consequently even the lightest metal link can be lengthened several thousand per cent without adding much to its weight.

The author has in his laboratory a show piece consisting of a bob of about 85 cc volume and a long chain of 50 mg links tied together with 2 inch lengths of No. 110 cotton thread, which gives in effect a chain with links 2.5 inches long. This arrangement is kept in a tall cylinder

full of water and serves as a convenient thermometer because the bob moves over a vertical distance of 18 inches with the ordinary daily fluctuations of temperature. It is an interesting experiment to cool the water in the upper part of the cylinder and watch this bob rise against the down flowing current of colder water.

The advantages of the chain balanced submerged float as a hydrometer are apparent at once if a comparison is made with the old type. (1) The position of the float with respect to a scale on the side of the carrying tube can be read far more accurately than the coincidence of the surface of a liquid with the scale on a projecting stem. (2) Since the whole equilibrium system is submerged, there are no troublesome surface tension phenomena to contend with and no unequal wetting of the projecting part. There is also no question but what the temperature of the whole apparatus will be that of the liquid in which it is submerged. These points permit the construction of a simple instrument of small size that will in a few minutes time give as accurate density determinations of liquids as can now be attained in several hours by means of a pycnometer.

There are also disadvantages inherent in the instrument. Since readings are made below the surface, the color of the liquid will interfere. The actual depth of liquid, however, through which the reading is made need not be more than a millimeter and therefore this is not a serious objection. Rather more liquid must be used for a determination than would be needed by other methods and finally the plan is of no use in liquids of very high viscosity.

In conclusion it should be stated that active work is in progress to study this chain balanced float with a view to developing (1) an instrument for the industries that will give accurate readings through four decimal places and (2) one for scientific investigations that, it is believed, will be accurate within a few units in the sixth place.

DEPARTMENT OF CHEMISTRY,
OHIO STATE UNIVERSITY.

THE IMPROVED MacMICHAEL VISCOSIMETER*

By Winslow H. Herschel

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1. Introduction

The MacMichael viscosimeter is a constant deflection, torsional instrument. Like all instruments of this class it has the advantage of speed of operation, while its weakness is due to variations in the torsional resistance of the suspending wire, which makes it difficult to obtain great accuracy.

As a somewhat primitive type of MacMichael viscosimeter has been previously considered[1] the description of the instrument will be confined mainly to the changes in design.[2] As shown in Fig. 1 the instrument consists essentially of a torsional pendulum with bob, *a*, hanging in the liquid to be tested. On a turntable, *b*, is placed an outer cup, *c*, concentric with the cup, *d*, holding the liquid, and the space between the cups contains water or oil and a coil of resistance wire for electric heating. As the turntable is rotated at constant speed by an electric motor, *e*, the pendulum has a constant deflection during any one test, due to the viscous drag of the liquid.

The deflection, which is a measure of the viscosity of the liquid, is read on a horizontal dial, *f*, concentric with the bob and supported on the hollow spindle, *g*, which encloses the wire. The circumference of the dial is divided into 300 equal parts, called MacMichael degrees or

* Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce.

[1] Winslow H. Herschel, Jour. Ind. and Eng. Chem., *12*, pp. 282, 818; 1920.

[2] Eimer and Amend, Bulletin No. 280, F. W. Shulenberger, Paint, Oil and Chemical Review, *72*, p. 10, 1921; W. M. Grosvenor, U. S. Patent No. 1 236 706; August 14, 1917.

M's.[3] Very small deflections may be measured by the observation, through a telescope, of a scale reflected in a mirror attached to the spindle. In order to bring the reading within the field of the telescope, when the deflection is considerable, the torsion wire is supported at the center of a rotatable notched disc, *h*, which may be secured in any one of 30 positions, 10 *M*'*s* apart.

The torsional pendulum is supplied with two bobs, one being similar to the disc used with the older type of instrument, and the other a cylinder 1 cm in diameter, and four centimeters high. When the latter is used, a third cup, *i*, is placed inside the other two, and oil or water is poured around it to complete the contact with the heating jacket.

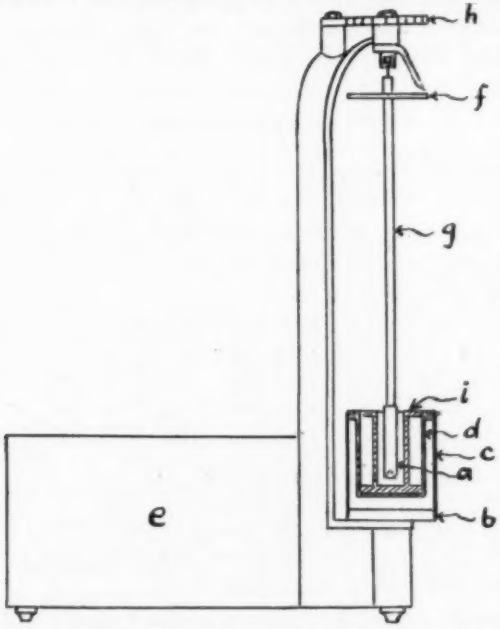

FIG. 1. *Schematic diagram of improved MacMichael viscosimeter.*

The use of a cylindrical bob changes the viscosimeter from a purely relative instrument to one more nearly absolute, that is, to one with which the viscosity may be calculated from the dimensions of the instrument and the deflection, without the use of calibrating liquids of known viscosity. It is the purpose of this investigation to determine

[3] One "*M*" is therefore a deflection of $\frac{2}{3}$ degree of arc, but its value in viscosity is a variable, depending upon the diameter of torsion wire, amount of turbulence, etc.

with what accuracy the MacMichael viscosimeter with a cylindrical bob may be used as an absolute instrument.[4]

2. Sources of Error

The constant deflection method was first suggested by Margules[5] who developed the simple formula but made no experiments. When there are two concentric cylinders, of which one is still and the other revolves with a constant angular velocity, the turning moment exerted on the stationary cylinder by the viscous drag of the liquid between the cylinders may be found by the equation

$$M = \frac{\pi^2 \mu\, h\, d_2^2\, d_1^2\, N}{30\,(d_2^2 - d_1^2)} = \frac{\pi^2 \mu\, h\, N}{30\, C} \qquad (1)$$

where M = turning movement in dyne centimeters.

μ = viscosity in poises[6]

h = wetted height of cylinders, in cm.

N = revolutions per minute of revolving cylinder.

d_1 & d_2 = diameters of cylinders (1 and 3 cm respectively in the MacMichael viscosimeter).

C = instrumental constant (8/9 with the above dimensions).

Equation (1) is rigidly accurate for infinitely long cylinders but takes no account of the resistance due to the ends. It has remained for subsequent experimenters to devise means for measuring M and methods of eliminating end effects as far as possible.

A. Means of Avoiding End Effects

Elie[7] and Zemplen[8] as well as Brodmann[9] (who also used cylinders) experimented with concentric spheres, but the method is not satisfactory because the necessary formula is approximate.

[4] The instrument used was kindly loaned for the purpose by Mr. C. G. Amend of Eimer and Amend.

[5] M. Margules, Sitzungsber. Akad. Wiss. Wien, **83**, part 2, p. 588; 1881.

[6] For purposes of calculation, the viscosity of a liquid may be defined as the force which will move a unit area of plane surface, with unit speed relative to another parallel plane surface, from which it is separated by a layer of the liquid of unit thickness. The term poise was first suggested by Deeley and Parr (Phil. Mag. **26**, p. 87; 1913) as the name for the cgs unit of viscosity, and they added:—"In the case of a soft solid (plastic substance) the so-called viscosity is not the same for all rates of shear, whereas the viscosity of a liquid is a physical constant and should be named." It is evident that Deeley and Parr never intended that the poise should be used as a measure of the consistency of plastic materials, and such use is to be deplored.

[7] B. Elie, Jour. de Physique, *1*, p. 224; 1882.

[8] G. Zemplen, Ann. der Physik, *29*, p. 869; 1909.

[9] C. Brodmann, Wiedemann Ann. der Physik & Chemie, *45*, p. 159; 1892.

Couette[10] used guard rings above and below the suspended cylinder. He ascribed his error of 15 per cent in the viscosity of water to the marked effect of slight eccentricity of cylinders when the clearance between them was only 0.25 cm. This method was however used with great accuracy by Gilchrist[11] and Harrington[12] with a clearance of 0.72 cm in finding the viscosity of air.

Brodmann used clearances of from 0.49 to 1.47 cm, and erroneously ascribed the lack of agreement between results with the different cylinders to slip at the boundaries, as suggested by Margules. End effects were eliminated by using the difference between the wetted heights in two different tests at the same speed, for h, and the difference in moments for M in equation (1).

Mallock[13] eliminated end effects in some cases by trapping air under the inner cylinder, and in other tests the water being tested floated upon a layer of mercury. Gurney[14] used the somewhat laborious method of measuring the difference in deflection of two totally submerged cylinders of as nearly as possible the same diameter.

The most accurate results with water appear to have been obtained by Drew[15] who, with clearances of 0.24 and 0.55 cm, found a viscosity of water at 20°C (68°F) of 0.01020 and 0.01010 poise, or only 1.5 and 0.5 per cent higher than the most probable value. It will be noted that the narrower clearance is about equal to that of Couette, and that the most accurate result was obtained with the wider clearance.

Drew's accuracy appears to have been due to his peculiar arrangement of guard rings shown in Fig. 2. He found that with the simple ring, C, the ratio of deflection to speed increased with the speed, and he reasoned that to eliminate end effects "the essential condition which must be fulfilled is that, when steady motion has been attained, all the points in the liquid which have a common speed must lie in a surface which, within the region considered, is everywhere equidistant from the bounding surface." The rings, R and S were accordingly added, as shown in the section at the right of Fig. 2, and "with this arrangement the results at different speeds showed no consistent difference." The surface of mean speed is indicated by the dotted lines. To what

[10] M. Couette, Annales de Chemie et de Physique, *21*, pp. 433, 448; 1890. A similar apparatus was used by E. Hatschek, Trans. Faraday Soc., *9*, p. 89; 1913.

[11] L. Gilchrist, Phys. Rev., *1*, p. 124; 1913.

[12] E. L. Harrington, Phys. Rev., *8*, p. 738; 1916.

[13] A. Mallock, Proc. R. S., *45*, p. 126; 1888 and Phil. Trans. R. S., *187*, p. 41; 1896.

[14] L. E. Gurney, Phys. Rev., *26*, p. 98; 1908.

[15] E. Drew, Phys. Rev., *12*, p. 114; 1901.

extent the air bubble trapped under the bob of the MacMichael instrument is effective in reducing or eliminating end effects could be determined only by very accurate experiment. It might be noted that the air bubble would be slightly compressed as the liquid to be tested was poured into the cup, and this would cause the liquid to rise inside the cavity at the end of the bob. A complete absence of turbulence and of end effects would be indicated if the deflection was proportional to the speed and to the depth of submergence of the bob.

B. TURBULENCE

While too small a clearance has been shown to cause danger of error from eccentricity, there is a danger that too wide a clearance

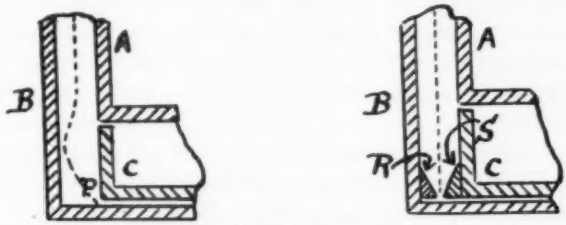

Fig. 2. *Arrangement of guard rings in Drew's viscosimeter.*

will lead to error from turbulence. In estimating the critical velocity at which turbulence begins, the best available equation is[16]

$$\frac{vb\,\rho}{\mu} = 2124 \qquad (2)$$

where v = relative velocity of cylinders, i.e. velocity of outer cylinder in centimeters per second.

b = clearance, or one half the difference in diameter of cylinders.

$$\frac{\mu}{\rho} = \text{kinematic viscosity} = \frac{\text{viscosity in poises}}{\text{density in grams per cc}}$$

Equation (2) gives 1918 for Couette's tests with water, and 1993 for his tests with air, which check the value of 2000 usually accepted as approximately correct for the corresponding criterion for critical velocity in pipes. Mallock's tests, with the outer cylinder revolving, showed values from 4180 to 32 350, but it should be remembered that equation (2) gives the minimum value at which permanent turbulence

[16] Lord Rayleigh, Phil. Mag., *28*, p. 619; 1914. He gives the constant as 177, which must be multiplied by 12 to give the value in the more usual units here employed.

may occur in long cylinders, and that higher speeds may be reached without turbulence under especially stable conditions.[17]

That the water in Mallock's tests was at times in unstable equilibrium, was shown by some interesting tests by Kelvin with Mallock's apparatus.[18] Kelvin found, at a speed where no turbulence appeared, that if turbulence was started by dipping a disturbing rod into the water, or by a sudden change of speed, the "smooth regime could only be re-established by slowing down and bringing up to speed again, gradually enough."

In other tests Mallock revolved the inner instead of the outer cylinder but found turbulence at all speeds. In Perry's tests[19] there was always the same cause for turbulence, though to a lesser degree, as he used an inverted cup with edge dipping into a circular trough, so that he had the combined effect of an inside and an outside cylinder as the rotating member. It is only with very viscous liquids, at moderate speeds, that a serviceable viscosimeter can be made with a revolving inner cylinder.[20]

A moderately wide clearance is an advantage, if not great enough to cause turbulence, because it reduces the influence of eccentricity and of variations in the clearance. Hersey[21] took advantage of this to construct a torsional instrument in which ordinary glass test tubes were used for the outer cylinder.

With the dimensions of the MacMichael viscosimeter, and with a kinematic viscosity of .010068 (as for example for water at 20°C) equation (2) gives a critical speed of 136 revolutions per minute. This does not, however, prove that the instrument can be calibrated with water, since in the similar case of the critical velocity of flow through short tubes, it was found that the shorter the tube the lower the critical velocity.

C. ERRORS DUE TO IMPERFECT ELASTICITY OF TORSIONAL WIRE

It does not seem practicable to use a bi-filal suspension in torsional viscosimeters because the depth of submergence of the bob must be

[17] For cases of flow in pipes where much higher values than 2000 were reached without turbulence, see Winslow H. Herschel, Trans. A.S.C.E., *84*, p. 537; 1921.

[18] Lord Rayleigh, loc. cit., p. 610.

[19] John Perry, Phil. Mag., *35*, p. 441; 1893.

[20] B. P. Veinberg and I. A. Smirnov, Chem. Abstracts, *6*, p. 3353; 1912. G. F. C. Searle, Proc. Cambridge Phil. Soc., *16*, p. 600; 1912. Kurt Molin, Proc. Cambridge Phil. Soc., *20*, p. 23; 1920.

[21] M. D. Hersey, Jour. Washington Acad. of Sciences, *6*, p. 526; 1916.

kept constant, and there is little agreement among experimenters as to the best form of suspension to use in order to eliminate errors due to semi-permanent set. Hogg[22] used a quartz fibre .0017 cm in diameter and 35 cm long, but most experimenters have used steel or phosphor bronze wires or ribbons of somewhat greater section.

Tomlinson[23] who appears to have paid the most attention to changes in torsional modulus of elasticity with time, found it necessary before taking readings to vibrate the pendulum through 10 degrees for about 1000 oscillations to reduce the internal friction of the wire to its permanent condition, "since long rest, or sometimes even a comparatively short rest, always raised sub-permanently the internal friction."

In the original MacMichael instrument steel piano wires were used, but in the directions accompanying the improved instrument it is stated that "for the most delicate work all parts of the suspended system should be non-magnetic—no steel—and for extremely low viscosities an especially light spindle may be secured." The objection to steel is not stated, and is not obvious.

The special spindle, above referred to, was not available in the tests to be described, but its need was demonstrated by the fact that the No. 40 wire (B. and S. gage) broke immediately from the weight of the spindle ordinarily used, and even the No. 36 wire broke after a moderate amount of use.

The directions also contain the following paragraph which shows that the semi-permanent set of the wire is as important a consideration in the constant deflection method, as in the method of decrement of oscillations as used by Tomlinson.

"At first thought these torsion wires are elastic and when a strain or twist is removed they should return to 0. Practically, however, no material is perfectly elastic. Under stress they become strained. Within the elastic limit part of this strain is immediately recovered and part of it remains as a temporary set from which they gradually recover on standing. For extremely accurate work the wires should be calibrated with respect to set, and readings corrected accordingly. For such accurate comparisons smaller readings should be made first and larger afterwards. A rest of 24 hours generally eliminates the set from previous work. For this reason also wires should be damped gently back to 0, if such minute corrections are to be applied, and not oscillated. Oscillation tends to eliminate set very quickly."

[22] J. L. Hogg, Proc. Am. Acad. Arts and Sciences, *40*, p. 611; 1904.
[23] H. Tomlinson, Phil. Trans. R. S., *177*, part 2, p. 771; 1886.

This quotation points to the most serious objection to the MacMichael or other torsional viscosimeter, and raises the question whether, if the above described precautions are taken, the speed of operation would be any greater than with an equally accurate efflux instrument, or, if these precautions were neglected, in order to get the same speed of operation as with one of the ordinary forms of technical efflux viscosimeters, the torsional instrument would give any more accurate results.

In the tests to be described it was assumed that set was negligible provided that the deflections of Table 1 were not exceeded, and that the graph showing variation in deflection with speed was a straight line or smooth curve. Any further refinement to correct for temporary set would make the instrument impracticable for routine tests of lubricating oils.

According to MacMichael there would be no permanent set with steel piano wire 0.0105 cm in diameter if a deflection of 600 M's was not exceeded, and Herschel and Dean's[24] table of allowable deflections is calculated with the same constant maximum stress in the wire, or constant value of

$$M'd = \frac{300\ \sigma\ l}{\pi\ G} = 16.0 \qquad (3)$$

where M' = deflection in M's

d & l = diameter and length of wire, in cm

σ = maximum stress in wire, in dynes per cm^2

G = torsional modulus of elasticity of wire, in dynes per cm^2

From equation (3), with $l = 25.4$ centimeters and $G = 880000$ kilograms per square centimeter (8.63×10^{11} dynes per cm^2), the stress is found to be 5820 kilograms per square centimeter (5.70×10^{11} dynes per cm^2). Table 1 is calculated from data of Eimer and Amend for composition wires.

While there are some irregularities in values of $M'd$ and of G as shown by Table 1 and Fig. 3, possibly due to deviations of the wires from the nominal diameters, $M'd$ was apparently intended to be one half as large as for steel wires, and G has an average value of about 433 000 kilograms per square centimeter (4.25×10^{11} dynes per cm^2). From these data the maximum stress is 1300 kilograms per square centimeter (1.28×10^9 dynes per cm^2).

[24] Winslow H. Herschel and E. W. Dean, Reports of Investigations, Bureau of Mines, Serial No. 2201; January, 1921.

TABLE 1. *Maximum allowable deflections for composition wires*

No. of wire	Diameter of wire, cm	Maximum allowable deflection, in *M*'s.
22	.0644	125.0
23	.0573	139.8
24	.0510	157.5
25	.0455	176.7
26	.0405	199.2
27	.0360	225.0
30	.0254	314.5
32	.0202	361.5
34	.0160	503.0
36	.0127	632.0
40	.0080	993.0

The problem of accurately measuring small moments in torsional viscosimeters is very similar to that met with in electrical instruments, where a spiral spring subjected to bending is ordinarily used and the

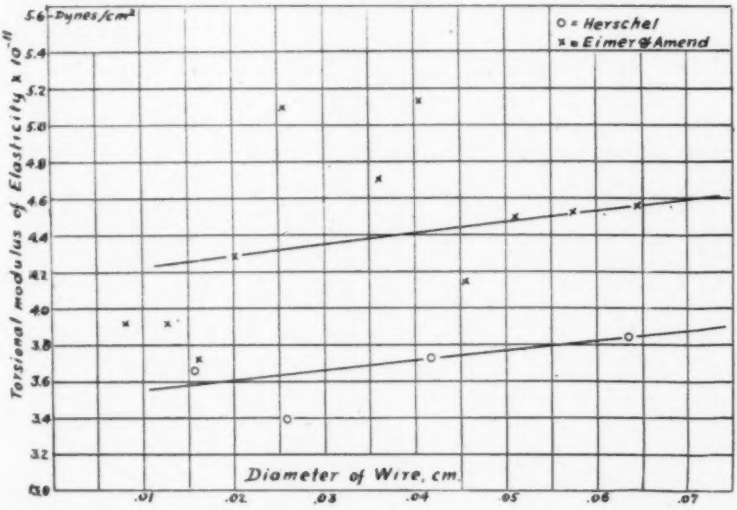

FIG. 3. *Variation in torsional modulus of elasticity with diameter of wire.*

stresses kept very low. Keinath[25] says that with good phosphor bronze the permissible bending stress may be taken as 500 kilograms per square

[25] G. Keinath, Die Technik der elektrischen Messgeräte, pp. 13, 152; 1921.

centimeter when it is not a case of precision but of technical instruments, but in general not over 300 kilograms per square centimeter should be used. Only with spring steel can a stress of 1000 kilograms per square cm be allowed.

There is an error due to semi-permanent set in spiral springs as well as in torsional pendulums, and even readjusting to zero does not get over the difficulty entirely. On this account the spring has been replaced by a weight in some instruments.[26]

Edgcumbe[27] in considering wattmeters constructed on the dynamometer principle, says,

"For instruments of the highest precision the spring usually takes the form of a cylindrical helix and is attached to a torsion head. When current is passing the suspended coil deflects against a stop, but is restored to the zero position by turning the torsion head so as to balance the deflecting torque against that due to the spring when twisted through a measured angle. A good example of a wattmeter of this class is that of Duddell."

Further details of the Duddell instrument are given by Keinath, who says,

"The sensitiveness is so high that with the highest permissible loading of the current and voltage coils (the latter 100 milliamperes) a complete turn of the torsion head with a scale length of 45 cm may be reached even with cos $\varphi = 0.1$, so that therefore measurements may be made very accurately even with low power factors."

It should not be supposed that the sensitiveness required in torsional viscosimeters is any less than in electrical instruments. Eimer and Amend give for the field of application of the smallest wire, No. 40, data showing a range of turning moment from 0.000119 to 0.1180 gram cm, while according to Agnew[28] the moments usually met with in D.C. ammeters are 0.214 to 0.873 gram cm, with larger values for other types of electrical measuring instruments.

3. THE MEASUREMENT OF MOMENTS

In the simplest method of measuring moments, either in taking viscosity measurements or in calibrating a wire, the moment to be measured is resisted by weights acting at the circumference of a pulley of effective radius, r.

[26] See Karl Schmiedel, Die Prüfung der Elektrizitäts-Zahler, p. 90; 1921.

[27] Kenelm Edgcumbe, Industrial electrical measuring instruments, p. 209, London, 1918.

[28] P. G. Agnew, Bulletin of the Bureau of Standards, 7, p. 48; 1911.

Then $$M = Pgr \tag{4}$$

where P = total weight in grams, usually the sum of two equal weights, so that the pendulum will not be deflected from the vertical.

g = acceleration of gravity = 981 cm per sec^2.

r = radius of pulley plus radius of thread suspending the weights.

This method was used by Couette with large moments, but he also used the more common method of measuring M by the angular deflection of the suspending wire.

In calibrating a wire, the modulus of elasticity is

$$G = \frac{5760\ Ml}{\pi^2 d^4 \theta} = \frac{4800\ M\ l}{\pi^2 d^4 M'} \tag{5}$$

where G = torsional modulus of elasticity in dynes per cm^2

M = turning moment in dyne centimeters

θ = angular deflection of wire in degrees.

Table 2 shows the results of calibration of the wires used in this investigation, calculated by equation (5) with values of $r = 0.556$ cm and $l = 27.9$ cm.

TABLE 2. *Torsional modulus of elasticity of suspending wires*

No. of wire, B and S gage	Diameter of wire, cm	$\frac{P}{M'}$	Torsional modulus in dynes per cm$^2 \times 10^{-11}$
22	.06350	.8446	3.846
26	.04170	.1516	3.724
30	.02580	.02026	3.389
34	.01570	.003047	3.661
36	.01270)	.....	(3.565)

The value of G for the broken No. 36 wire was estimated from Fig. 3 which shows also values of G calculated from data of Eimer and Amend for a typical set of composition wires. The modulus of brass wire is given by Watson[29] as 3.8×10^{11} which is in approximate agreement with Table 2. It does not appear to have been previously observed that G varies with the diameter.

In another method often used for calibrating wires, the inner cylinder and suspending wire form a torsional pendulum, the time for a complete oscillation being

$$t = \frac{8}{d^2}\sqrt{\frac{2\pi l W\ (d_1^2 - d_0^2)}{G}} \tag{6}$$

[29] W. Watson, A Text-book of Physics, p. 204; 1902.

where W is the weight of a hollow cylindrical bob in grams, and d_1 and d_0 are its outer and inner diameters. Poynting and Thomson[30] say that the values of G as found by equation (6) are, as a rule, higher than those found by equation (5), and this tendency would be accentuated if, with the MacMichael viscosimeter, the cavity to trap the air bubble was neglected in calculation, and d_0 taken as zero.

According to Hersey it is imperitive that t be small, and his own viscosimeter has a calculated value of 6 and observed value of 5 seconds. The viscosimeter having a steel wire 0.0266 cm in diameter showed by experiment, with the disc bob, a value of 6.82 seconds, and from these data, with the help of equation (6) the probable maximum time with a disc bob, with a composition wire 0.0127 cm in diameter would be 46.5 seconds. With the same wire, using the cylindrical bob, the estimated time would be 5.6 seconds. In tests with the smaller wires considerable difficulty was experienced in obtaining readings because the pendulum was always in oscillation.

By combining equation (1) with (5) or (6) the viscosity may be calculated directly from the observed data. From equation (5)

$$\mu = \frac{30\,C\,Pgr}{h\,N\,\pi^2} = \frac{C_1 M'}{h\,N} \tag{7}$$

or from equation (6)

$$\mu = \frac{C\,W\,d_1^2\,M'}{10\,l^2\,h\,N} = \frac{C_1\,M'}{h\,N} \tag{8}$$

where C_1 is an instrumental constant which varies with the dimensions and torsional modulus of elasticity of the wire.

Values of C_1 and of the number of poises represented by a deflection of one MacMichael degree were calculated, with the help of equation (7), from data of Table 2, and results are shown in Table 3. It was assumed that the submergence of the pendulum bob was 4 cm and the speed 19.1 revolutions per minute. This speed is considered by the designer of the instrument as standard, but it is doubtful whether there is any particular virtue in this particular speed as the rate of shear of the material under test varies with the distance from the center of rotation.[31]

According to Hersey, if a torsional viscosimeter has a completely immersed cylinder, so slender that the end effects are small, and running so slowly that the final deflection is independent of the density,

[30] J. H. Poynting and J. J. Thomson, Properties of Matter, p. 83; 1903.

[31] Winslow H. Herschel, Paint, Oil and Chemical Review, *72*, p. 10; October 5, 1921.

only a single calibration constant is required, namely θ_0 the deflection at a speed of one revolution per second in water at 20°C (68°F), which it is desirable to have large.

TABLE 3. *Theoretical relation of absolute viscosity to deflection for different sized wires*

No. of wire	C_1	Poises per MacMichael degree	
		Eimer and Amend, for set of "typical wires"	Herschel, for wires of Table 2.
22	1291.8	20.50	16.93
26	224.66	3.68	2.942
30	30.033	0.560	0.393
34	4.564	0.0640	0.0598
36	1.852	0.0266	0.0242

As previously stated it was found that the No. 36 wire was the smallest which could be used without the special spindle, and it may be found from equations (5) and (7), with $h=4$, the maximum available speed of 120 revolutions per minute, and a viscosity of 0.01005 poise, that the maximum deflection theoretically attainable would be 3.13 degrees. Expressing this result in terms of $\theta_0=1.57$ degrees (1.3 M's) it may be compared with Hersey's calculated value of 18.4 degrees (15.3 M's) and observed value of 21.8 degrees (18.1 M's) for his own instrument.

It is evident that the MacMichael viscosimeter has so small a value for θ_0 that it can not be calibrated with water when deflections are read from the graduated dial.

4. EXPERIMENTAL RESULTS WITH DIAL READINGS

If C_1 and μ are known, in equations (7) and (8), and h is regarded as unknown, then

$$h+k=\frac{C_1 M'}{\mu N} \tag{9}$$

where k is the correction due to end effects not entirely eliminated by the air bubble under the pendulum bob, and h is the nominal submergence of the bob.

In Fig. 4 are shown results obtained with the cylindrical bob with 2 and 4 cm submergence and oils of various viscosities. The numbers on

lines show viscosities in poises, as determined by the Bingham viscometer[32] and the submergence in centimeters.

By the use of equation (9) it was determined that k had an average value of 0.12 centimeter for tests with the No. 34 wire; with the No. 30 wire the results were less consistent. Since for a given viscosity and submergence the deflection should be proportional to the speed, the theoretical graphs would be straight lines passing through the origin and through the calculated points indicated on Fig. 4.

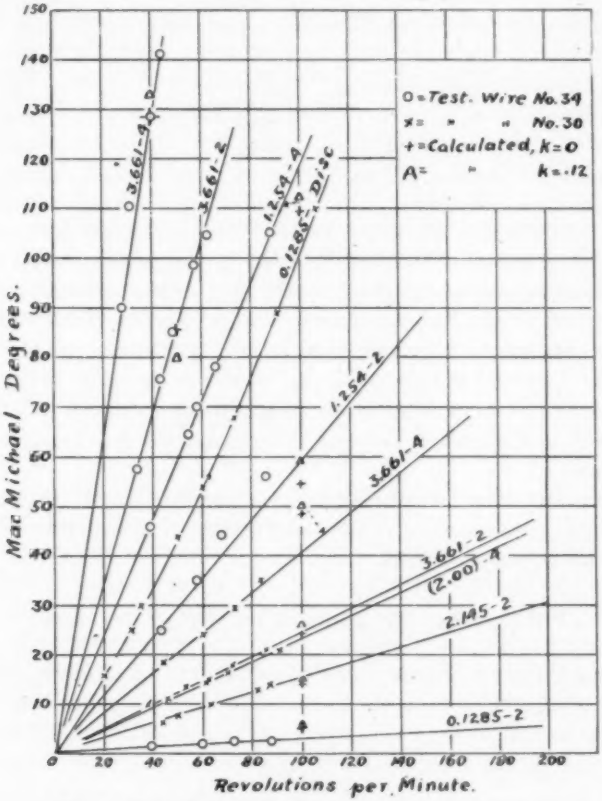

FIG. 4. *Determination of end effect by variation of submergence.*

It does not appear at all certain that the apparent end correction is not due to error in filling or to error in determining C_1. In any

[32] Winslow H. Herschel, J. O. S. A. & R. S. I., *6*, p. 875; 1922.

event it is apparent that the effect of k, whether due to filling error or to end effect, is small in comparison with the possible error due to temporary set of wires, and might be decreased by the use of longer cylinders.

Fig. 5 is similar to Fig. 3 but the submergence is 4 cm in all cases and a greater number of wires was used. In this case k has been assumed negligible, and C_1 has been calculated from data of Figs. 4 and 5, as shown in Table 4.

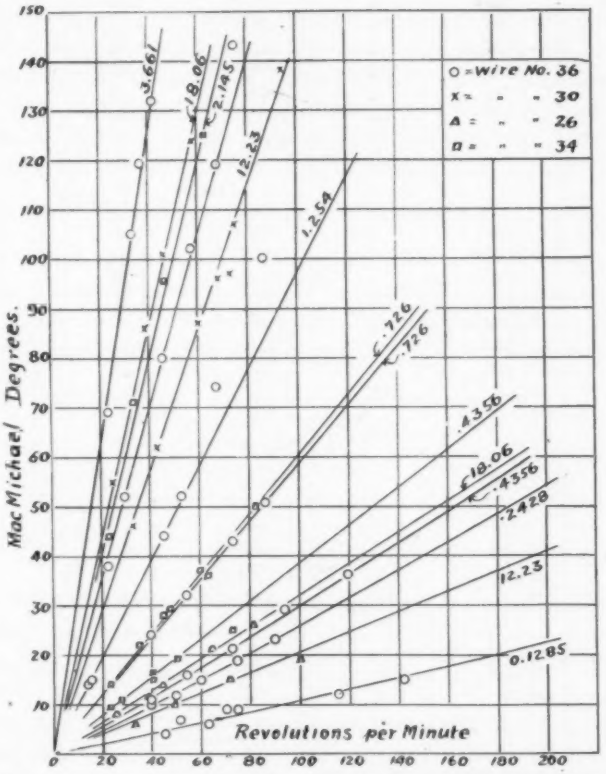

FIG. 5. *Dial readings with various viscosities and wires of different diameters.*

Comparing the last two columns, it is seen that the No. 36 wire must have been of approximately the same size as the No. 34 wire, and consequently much larger than assumed. The average value of C_1 found for the No. 34 wire by calibration with liquids, agrees within

three per cent of the value calculated from known moments. The agreement with the No. 30 wire is less satisfactory.

TABLE 4. *Constants of wires obtained by calibration with oils of known viscosity*

Wire No.	Viscosity, poises	Submergence, centimeters	$\frac{M'}{N}$	C_1 by calibration	
				With liquids of known viscosity	With known moments
36	.1285	4	.1110[x]	4.63	(1.852)
	.243	4	.2500[x]	4.11	"
	.436	4	.300[x]	5.82	"
	.726	4	.590	4.92	"
	1.254	4	.976	5.15	"
	2.145	4	1.752	4.90	"
	3.661	4	3.146	4.66	"
			Average	4.91	
34	0.1285	2	.0278[x]	9.25	4.564
	.436	4	.3857	4.53	"
	.726	4	.6060	4.79	"
	1.254	4	1.193	4.21	"
	1.254	2	.591	4.25	"
	2.145	4	2.000	4.29	"
	3.661	4	3.207	4.57	"
	3.661	2	1.717	4.26	"
			Average	4.43	
30	(2.000)	4	.2332	34.3	30.033
	2.145	2	.1538	27.9	"
	3.661	4	.4085	35.8	"
	3.661	2	.2450	29.9	"
	12.23	4	1.438	34.1	"
	18.06	4	2.168	33.3	"
			Average	32.2	
26	12.23	4	.2067	237.0	224.66
	18.06	4	.3202	224.8	"
			Average	230.9	

[x] = approximate: graphs slightly curved.

5. MEASUREMENT OF DEFLECTION BY THE MIRROR AND SCALE METHOD

As noted in Table 4, oils of low viscosity gave curved graphs which indicated turbulence. There remained however the possibility that the

smaller wires might be calibrated with water by using the lowest attainable speed of 10 revolutions per minute to avoid turbulence, and the mirror and scale method to get a readable deflection at such a low speed. According to Eimer and Amend's table of the range of viscosities for the different wires, this could only be done with the No. 40 wire and an especially light spindle.

The method of measuring small angles by a mirror and scale, often applied to galvanometers, may be applied to torsional viscosimeters when great accuracy is desired. From the method of supporting the torsion wire from a notched disc, the maximum angle which must be read on the scale is about 12 degrees, but this is so large that there would be considerable inaccuracy in the assumption, convenient with very small angles, that the angle is proportional to the tangent. If however a curved scale is used, the deflection in radians is equal to $\frac{\Delta}{2D}$ where Δ is the difference in scale readings, and D is the distance of the scale from the mirror.

When a straight scale is used, as in the tests to be reported, there may be two cases, (1) where the deflection is very small, or (2) where the angle is too large to assume that it is proportional to its tangent. For case (1),

$$\mu = \frac{75C_1}{\pi hND} = \frac{C_2\Delta}{hND} \tag{10}$$

or for case (2), since $\theta = \frac{90\Delta}{\pi D}$

$$\mu = \frac{5C_1\theta}{6hN} = \frac{C_3\theta}{hN} \tag{11}$$

where θ is the deflection in degrees, and must be calculated from the relations

$$\frac{d}{D} = \tan\beta \qquad \frac{d'}{D} = \tan(2\theta+\beta)$$

where d and d' are the scale readings before and after deflection. It is convenient to eliminate β by having the zero of the scale in line of the telescope, perpendicular to the mirror before deflection.

Fig. 6 shows results obtained by the mirror and scale method using water at 23°C (73°F) and an oil of 0.1285 poise viscosity at 25°C (77°F), the dotted lines showing the theoretical deflection as calculated from equation (11). It will be seen that these dotted lines are tangential to the experimentally determined graphs, drawn in full, and that

there is apparently only a very short length, if any, of the experimentally determined graphs which does not show the curvature due to turbulence. Thus even if the readings could be sufficiently magnified at very low speeds, it would be impossible to make an exact calibration of the wires by the use of water. In fact the graphs obtained with light oils, both on Figs. 4 and 5, show that a calibrating liquid should be used having a viscosity of about 0.5 poise at least.

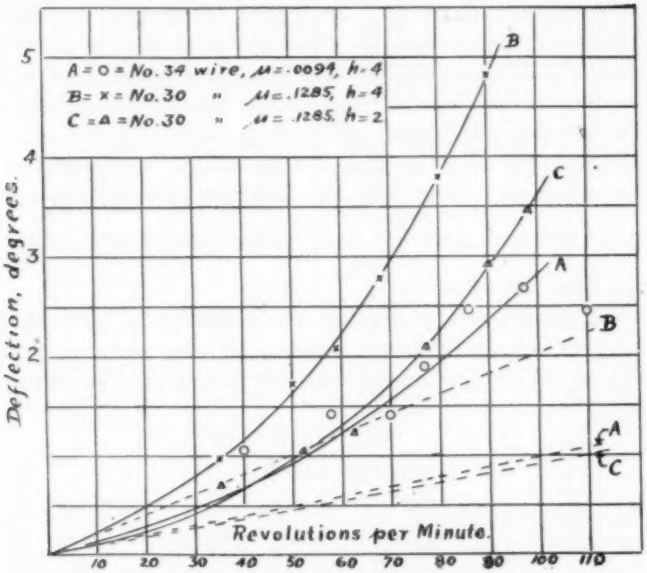

FIG. 6. *Determination of deflection by the mirror and scale method.*

The turbulence could be reduced by decreasing the clearance between the cylinders, but it is doubtful whether this could be done without the use of more than one cylindrical bob, without making the instrument less suitable for tests of plastic material. Theoretically the clearance between the cylinders ought to be increased as the size of the particle is increased, so that, if the outer cylinder is kept of constant diameter, a larger inner cylinder should be used with true liquids than with plastic materials.

6. CONCLUSIONS

1. The wires of the improved MacMichael viscosimeter can not be calibrated with water, on account of turbulence, and a more viscous liquid, with a viscosity of about 0.5 poise is required.

2. The end effects are negligible in comparison with errors due to semi-permanent set of the torsion wires.

3. The MacMichael viscosimeter might be greatly improved if other means than the torsion wire were provided for measuring the moment due to viscous drag, if this could be accomplished without the introduction of more serious errors due to friction. The problem is comparable to the measurement of moments of similar magnitude in electrical measuring instruments.

Bureau of Standards,
Washington, D. C.

An Electron Tube Amplifier Using 60-Cycle Alternating Current to Supply Power for the Filaments and Plates.—Electron tube amplifiers form an important part of practically all radio receiving sets, except the most simple types. For good operation, amplifier tubes require for the filament a source of voltage of very constant value (usually about 6 volts) and for the plate a source of voltage of from 40 to perhaps 300 volts. The development of an amplifier which can be supplied from the ordinary 110-volt ac lighting mains is therefore of considerable practical importance. Such an amplifier using five stages of amplification has been developed at the Bureau of Standards. There are three stages of radio-frequency amplification and two stages of audio-frequency amplification.

The voltage for the filaments is obtained from one winding of a 60-cycle transformer having four windings. The primary is connected to the 110-volt 60-cycle lighting supply. The grid circuits, instead of being connected directly to one terminal of the filament supply, are connected to an intermediate point between the filament supply terminals through the medium of a slide-wire voltage divider, the ends of the voltage divider being across the filament supply and the grid circuits being connected to the sliding contact through a 1 microfarad condenser shunted by a 1 megohm leak resistance. One voltage divider is used for the grids of the radio-frequency tubes and one for the grids of the audio-frequency tubes. By adjusting the sliders of the voltage divider, the grids may be brought to a value of voltage which is constant with respect to the changing voltage of the filament, thereby reducing the ac hum. The 1 microfarad series condensers allow the grids to assume a favorable condition for amplifying. It was found more desirable to replace the condenser and leak resistance in the grid leads of the audio-frequency tubes by a 10-volt negative dry battery which gave better amplification with less hum.

A crystal detector gave much better operation with less hum than a tube detector because the former can be entirely insulated from the ac power circuit and picks up less hum than the tube detector.

Plate voltage is supplied from another winding of the 60-cycle transformer, the ac voltage being rectified by means of a two-electrode electron tube and smoothed out by large condensers. The paper gives diagrams of various circuits used in the course of development and the final circuit employing five amplifier tubes.

This latter gives good amplification, with a slight residual hum which is not great enough to be objectionable when receiving signals of ordinary readable strength. The residual hum is of course more objectionable when extremely weak signals are being received. Radio telephone music and conversation are clearly reproduced. [P. D. Lowell. Bureau of Standards Sci. Paper No. 450.]

R. S. OULD

FIXED POINTS FOR REFRACTING AND SPECTACLE FITTING

By Sidney L. Olsho

American lens manufacturers are constantly aiming to establish a more perfect correspondence between the "effective power" of test case lenses and that of commercially supplied ophthalmic lenses. Coincidentally certain elementary principles are disregarded in all the trial frames and refracting apparatus on the market.

For example, a painstaking refraction is made with any of the commonly used trial frames. In the construction of these frames the manufacturers proceeded on the theory that all faces are symmetrical. During the entire refraction therefore the position of the trial frame may be extremely faulty. The axis readings are accepted because the trial frame has perhaps been in the shop recently and pronounced "trued up." The refraction may be followed by a most careful adjustment of spectacles or eye glasses. The adjustment takes into account the natural asymmetry of the face. At this point the very exercise of care introduces a vital discrepancy between the lenses as used in the refracting room and the lenses supplied the patient, the discrepancies disturbing not only the "effective powers" but also diverting the cylinder axes.

In trial frames recently placed on market, numerous adjustments are featured. None of these, however, solves the problem of making trial lens and ophthalmic lens positions correspond. For instance, a trial frame adjustment which permits the lowering of one trial cell independently, is particularly confusing. It is obvious that refractions must be made with the trial lenses in that position before the eyes which accurately corresponds to and predetermines the position of the ophthalmic lenses to be worn. In other words there must actually be a co-ordination of refraction with spectacle and eye glass fitting.

The statement is confidently made that this can be done only by the adoption of the system of *Base Line Refraction*, using *Fixed Points* for both the adjustment of the trial frame and for the adjustment of the finished spectacles and eye glasses, and advocated particularly in connection with the use of the trial frame devised for that purpose and

illustrated herewith. The essential features of the trial frame are enumerated:—

1. The front is rigid and adjustable to interpupillary distance.

2. A horizontal test rod rests on four supporting pins and crosses the four marks indicating the 180 degree meridian of the trial frame. It is used to verify the alignment of the trial frame front.

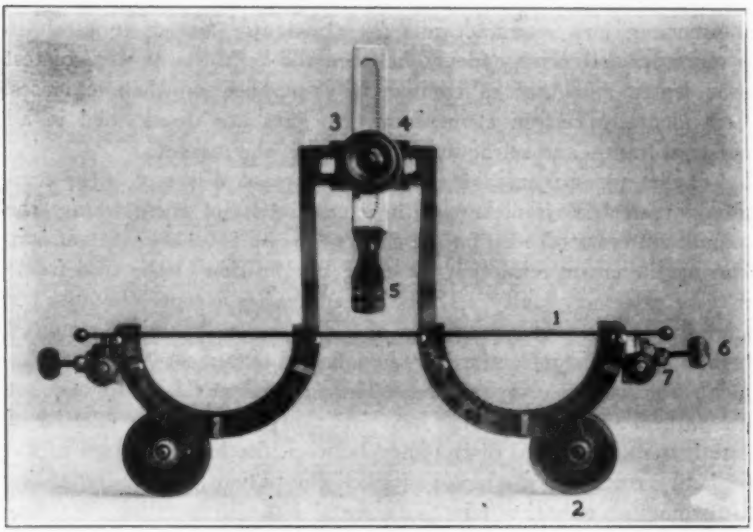

FIG. 1. (1) The 180 degree markings appear on both temporal and nasal sides for each eye, all four on one continuous line, as shown by the long horizontal rod resting on four supporting pins. (2) Grooved friction wheel to rotate trial cylinders. (3) Wheel for centering trial cells. (4) Wheel for raising or lowering perforated, conforming nose rest. (5) Set screw for positioning nose rest forward or back. (6) Knob, one for each temple, controlling its spread, securing even tension on both sides of the head and equal distance of both cells from the eyes. (7) Knob for delicate control of the new, separately adjustable tilting temples. Tilt one temple only and the corresponding end of the trial frame is raised. Tilt both equally and the trial frame is tilted forward from the top to the desired angle.

(Few trial frames fronts would be found in alignment were their 180 degree meridians marked at all four points, which of course in the majority of trial frames is not the case.)

3. Two centrally-beaded short horizontal rods replace the single rod when the trial frame is on the patient's face. The trial frame is so positioned that the two rods are each exactly opposite an external canthus.

(The eye balls are not fixed points. The pupils are therefore not stationary. They cannot serve constantly and invariably as landmarks for a fixed base line or 180° meridian for both the trial frame and the finished spectacles or eye glasses. They cannot serve because their positions are altered by every movement of the head, because ocular deviations and nystagmus are common, because pupils are often irregular, unequal and displaced. But the external canthi *are* fixed

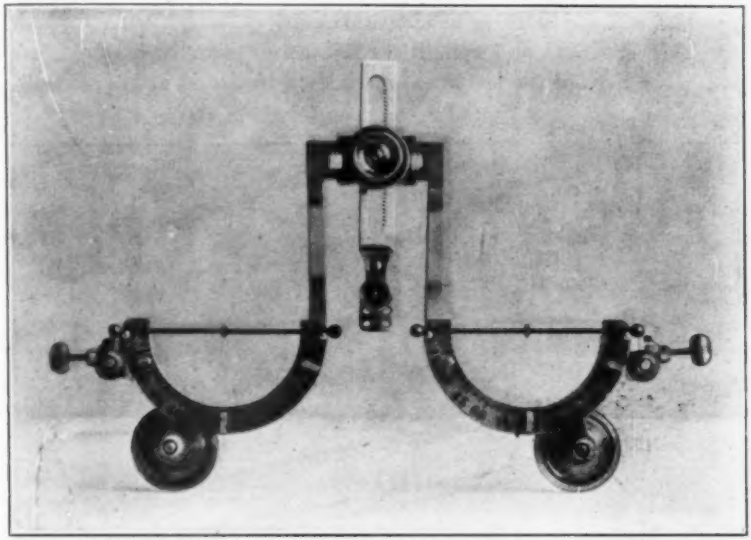

FIG. 2. Each one of the short horizontal rods should be opposite an external canthus when frame is correctly levelled on the patient's face. When one ear is higher, or the face asymmetric, it becomes necessary to tilt one temple more than the other to bring the continuous horizontal line of the trial frame front opposite and parallel to the line extending from external canthus to external canthus. When both temples are tilted equally the plane of the trial frame front is tilted forward from the top to the desired angle.

points, than which there are none better. They serve to determine the extremities of a Base Line even though one eye has been enucleated.)

4. The trial frame temples are straight. They are delicately, yet firmly and separately adjustable as to tilt and lateral pressure. Tilting both temples equally tilts the plane of the trial lenses. Tilting one temple separately raises the corresponding temporal end of the trial frame front.

(There are other trial frames with tilting temples but they are not delicately and separately adjustable nor devised for the purpose of bringing a fixed, straight, marked, base line of the trial frame opposite to, exactly parallel with and at the same height as another straight line on the patient's face, a line determined by two lateral *fixed* points. They are not adjusted in relation to fixed points hence it is not probable that they will be adjusted in exactly the same position at two different sittings by the same operator, nor probable that two different operators will adjust them in identical positions.)

5. Thin spherical and cylindrical test lenses, exactly centered, are used. The axes of the latter are marked *on the lenses* at *geometrically*

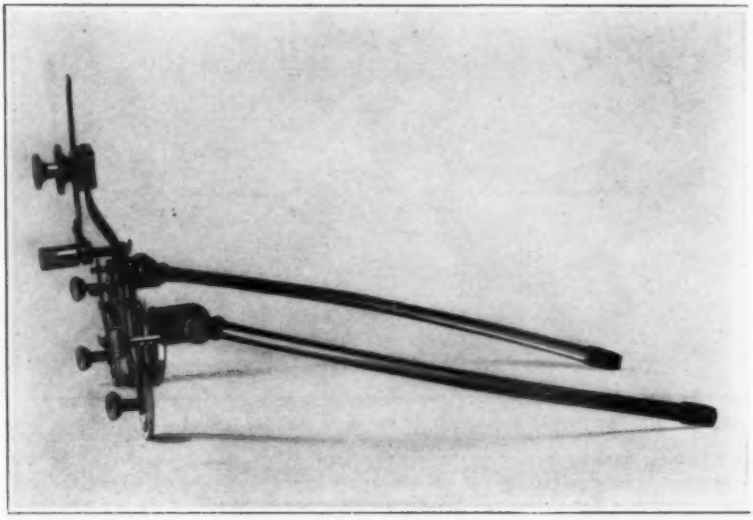

FIG. 3. *Tilting both temples equally tilts the plane of the frame front.*

opposite points. When placed in the trial frame at the 180 degree axis both marks on each cylindrical trial lens are seen to hit off correctly with two 180 degree marks on the trial frame, thus verifying the markings each time a cylindrical test lens is placed in the trial frame.

(The cylindrical test lenses in trial cases throughout the length and breadth of the land will be found marked on their axes but not with any degree of constancy at *geometrically opposite points*. Axis readings with such cylinders cannot attain accuracy. The two marks are not at the same axis.)

6. The trial lenses are large, the diameter of the cells being 38 mm. (The use of a small cylindrical test lens is equivalent to working with a very short arc. The movement of such a lens over a five degree arc is a very minute excursion. The movement of five degrees of the larger lenses is an appreciable and *discernible* excursion.)

With the aid of the delicate device for tilting the temples separately and the short horizontal test rods in place, the straight line crossing the four 180 degree markings on the axis scales of the trial frame is

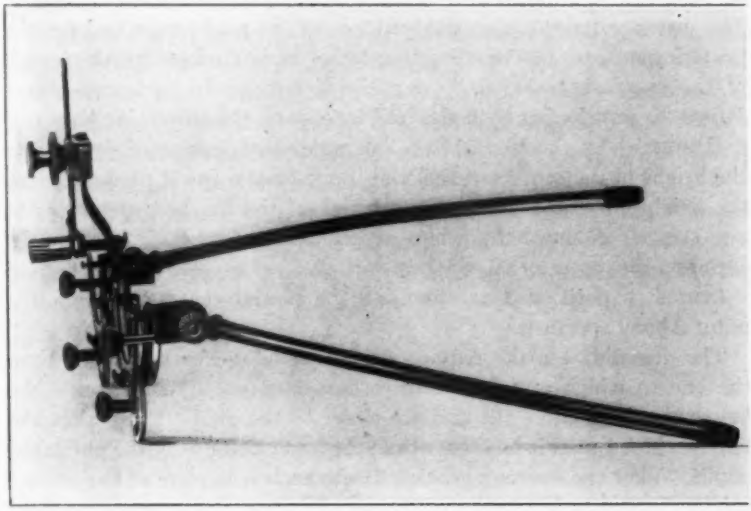

FIG. 4. *Tilting one temple independently raises the corresponding temporal end of the trial frame front, maintaining the correct level in the presence of one low ear.*

brought exactly opposite to a straight line determined by the two external canthi of the patient. Subsequently a pair of spectacles or a pair of eye glasses is so adjusted that a straight line temporarily marked on the horizontal meridians of the correcting lenses is brought into the identical relation to the external canthi of the patient.

The trial frame adjustment just mentioned is made while the trial cells are very close in to the eyes and the short test rods in place. Having levelled the trial frame front in this manner and at this height, bringing the short rods each directly opposite an external canthus, next, by means of the set screw on the nose rest, the trial frame front is moved off from the eyes the required distance.

DISTANCE FROM THE CORNEA:—Refractions are made with the trial frame front just so far away from the eyes as to permit the lashes to

escape touching the posterior surfaces of the lenses which are in the posterior cells of the trial frames. This distance is not constant. There are many high myopes who may have only a very few short eye lashes. It is an advantage to have their refraction done with lenses very close to the eyes, provided that the correcting lenses are placed in a corresponding position. On the other hand some lashes are so long that it is necessary to have the lenses farther than 14 mm from the corneas. The effective focal powers of both trial lenses and correcting lenses will correspond, since the distance of both from the corneas coincide. The distance for the posterior surfaces of the trial lenses and for the posterior surfaces of the correcting lenses is determined by the length of the eye lashes. Their extremities determine in each case *Fixed Points* for positioning both the trial lenses and the correcting lenses.

HEIGHT:—The horizontal base line previously designated determines the height of the centers of the trial frame lenses, and it predetermines the height for the centers of the lenses of the finished spectacles or eye glasses, as shown by white pencil lines across their horizontals, each opposite an external canthus.

Lenses so positioned are found to be placed centrally before the natural bony aperture.

The demands I make require a position of the lenses higher than the one to which opticians seem accustomed to adjust lenses. My lenses also have more tilt and are closer to the eyes. I find that the line to which I refer is no higher than the lower margins of the undilated pupils. View the wearer's profile. If spectacle lenses are at the correct height the spectacle temple will be seen to cross the external canthus, and this is the height at which the refraction is to be done.

By insisting on this high position I have given innumerable patients that comfort which they have constantly sought for themselves, by habitually pushing. their glasses upward; I have contributed greatly to the improvement of their personal appearance with glasses and have permitted them to use their eyes, most nearly in the primary position.

Lenses for reading or close work exclusively, may be slightly lower, but not much. Patients bring the depressors of the eye ball into action much more while the eyes are being tested than at other times. Ordinarily a book or paper is slightly elevated. Observation will show that for any extended close work the individual inclines head, neck and back sufficiently so that the eyes may be used in the position of greatest ease, namely as nearly as possible in the primary position (undepressed).

TILT:—The refraction is done with the planes of the trial lenses tilted at an angle parallel to the orbital aperture. (i.e. to a line extending from the center of the upper, sharp, thin, bony rim of the orbit to a corresponding point on the lower sharp, thin, bony orbital rim). Subsequently a pair of spectacles or eye glasses is adjusted so that the lenses have the identical tilt.

One cylindrical trial lens should never be moved independently of its fellow to a lower parallel. Contrary to popular conception this brings about an almost hopeless inco-ordination as is pointed out by a diagram (Fig. 7) in my paper "The Co-ordination of Refraction with Spectacle and Eye Glass Fitting"—(Am. Jour. Oph., July, 1920). If a patient has been refracted with one trial cell dropped, and a pair of frameless spectacles then made, on that prescription, with lenses centered and their four screw holes on one line, the axis even though it be 90° or 180° will be diverted.

The essence of the correct procedures recommended resides in the fact that there is in every case refracted and fitted by this method a perfect correspondence between the positions of the trial lenses and the correcting lenses worn by the patient.

PHILADELPHIA, PA.

A New Method for Studying Ionizing Potentials. The difficulty of interpreting critical potentials above the 1st resonance, and 1st ionization, led the author to apply positive-ray analysis to the products of a mercury discharge tube. The main trouble is to obtain a detectable number of ions and still to prevent collisions while the resulting positive ions travel through the analyzing fields. These conditions were secured by passing a unidirectional stream of vapor through the tube in a direction perpendicular to the axis (along which the potentials were applied), the remainder of the tube being maintained at a high vacuum. Wide slits were used in the analysis chambers to secure large currents; therefore high resolution was impossible. At a given accelerating voltage, the magnetic field of the ion-analyzer was varied. Thus ions with different values of m/e fell into the detecting chamber, and the resulting current was plotted against magnetic field. The voltages at which new peaks appeared in the curves were thus determined. Hg^{++} ions were detected at 19 ± 2 volts, and were presumably due to double impact. The relative magnitudes of the peaks due to Hg^{+} and Hg^{++} with low electron currents lead to the conclusion that Hg^{++} is produced by single impacts at higher voltages. Attempts will be made to apply the method to gases. [H. D. Smyth, Proc. Roy. Soc. A., *102*, pp. 283-293, 1922.]

ARTHUR E. RUARK